

Retirement Income Accumulation: Non-Gaussian Analysis of Accumulation Strategies and Products

A White Paper from Aftcast.com

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Retirement Income Accumulation: Non-Gaussian Analysis of Accumulation Strategies and Products

Executive Summary:

In this paper, we analyze the **Capital Accumulation Efficiency (CAE)** of various strategies, including, asset allocation strategies, variable annuities, fixed index annuities and other similar products using non-Gaussian approach. CAE measures how effectively a method, a strategy or a product measures against an optimized investment portfolio for accumulation purposes.

First, we calculate the savings required to provide a fixed dollar accumulation target with 90% certainty for the investment portfolio with varying asset allocations and time horizons. We establish the optimum asset mix for each time horizon using our non-Gaussian approach. This becomes our **base case**.

Then we compare the capital required for this base case to various **test cases**:

1. Portfolios with various values of alpha, fixed income yield premiums, and real return of inflation indexed bonds, rebalancing thresholds and rebalancing frequencies.
2. Portfolios with different asset allocation strategies, such as age based and target date.
3. Different Fixed Index Annuity products with the interest credit linked to equity index (FIA).
4. Different Variable Annuity products with guaranteed minimum withdrawal benefits (VAGMWB)

We define the CAE as the ratio of capital required in the base case to the capital required for the test case expressed in percentage. For example: The base case indicates that you need \$320,000 as initial capital to generate \$1 million after a 30-year time horizon with 90% certainty. The test case using a certain FIA indicates that you need \$240,000 to achieve the same objective. Then the capital efficiency of this particular test case is 133%.

If the capital efficiency is above 100% then the test case is the more efficient way of accumulating assets for retirement. If the capital efficiency is below 100% then an investment portfolio at the optimum asset mix is the more efficient way of accumulating assets for retirement.

Throughout this paper, we avoid using any deterministic and Gaussian forecast methodology, such as assumed future growth rates, future inflation rates, or any type of Monte Carlo simulators. Instead we use the aftcasting methodology that uses the actual market history. Aftcasting reflects the actual sequence of events, the actual sequence of returns (stocks, interest rates and inflation), actual correlation between stocks, interest rates and inflation; and finally, the actual volatility as they occurred since 1900. Aftcasting methodology and calculation tools for writing this article was developed by the author of this paper.

We did not include any tax consequences in our analysis. We only looked at actual portfolio performances, excluding any tax advantages of some specific products, strategies or account types.

Designing a retirement accumulation strategy with a 90% certainty does not reflect of the author's future expectations of market performance. Rather, it provides a better-engineered, purpose-driven design that allows the investor to achieve his dollar target within his specific time horizon. By definition, designing for a 90% certainty means that the investor will reach his dollar target before his planned time horizon in 90% of the time –based on market history. When that happens, he can then invest surplus assets as described in the addendum at the end of this paper. At that time, the purpose of investing switches from accumulating a target dollar amount to maximizing growth at optimum risk.

Summary of Findings:

The non-Gaussian aftcast of all years since 1900 shows that:

1. The optimum asset mix between equities and bonds points to a much lower equity content in a non-Gaussian approach when compared with the Gaussian approach (efficient frontier, MPT etc).
2. The performance of bonds is just as important as the performance of equities in an accumulation portfolio.
3. Target-Date funds are much more likely to miss a target than a portfolio with a fixed, optimum asset mix.
4. Fixed index annuities that are of the annual-point-to-point type, have a significantly better chance of achieving a specific future target dollar amount than a portfolio with a fixed, optimum asset mix.
5. If the purpose is to accumulate within the next 10 to 15 years for a lifelong retirement income, variable annuities with step-up features generally have the highest capital accumulation efficiency.

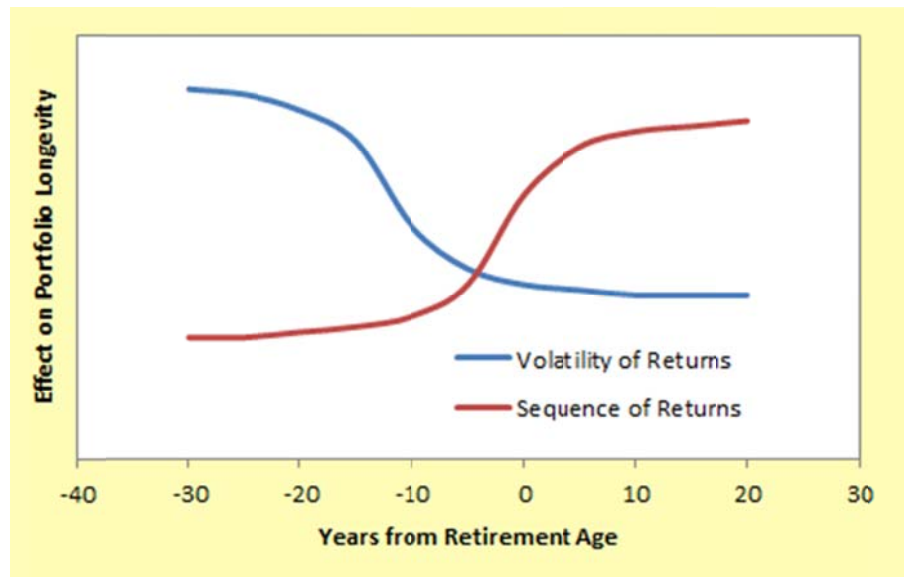
Introduction:

One of the challenges of accumulating retirement assets is to save sufficient funds in a limited time horizon. The retirement assets must be large enough to finance retirement, as well as other factors such market risk, longevity risk and inflation risk.

The most important determinant of portfolio longevity during retirement is the withdrawal rate. The second most important determinant is the sequence of returns¹. This is followed by the inflation factor. After that it gets a little blurry; the fourth place is a tie between “asset selection and monitoring” and “asset allocation”.

Asset allocation is an important tool to control the volatility of returns during the early years of the accumulation stage. However, it does very little to control the sequence of returns. Hence, as one gets closer to retirement, the well-publicized importance of asset allocation diminishes significantly and it is replaced by the importance of sequence of returns. Figure 1 depicts the relative importance of these two factors on portfolio longevity of a retirement portfolio for withdrawal rates over 3.5%.

Figure 1: The Effect of volatility of returns and sequence of returns on portfolio longevity over retirement life cycle.



¹ Otar, Jim, “Unveiling the Retirement Myth”, 2009, Chapter 31 “Determinants of a Portfolio’s Success”

When saving for retirement, one of the important questions is “what strategy or product can help accumulate a specific target amount of retirement assets with the highest efficiency and certainty?” It is important to know, for example, \$500,000 invested today, can grow to \$1 million over the next twenty years with some degree of certainty. For that, we turn to aftcasting.

Aftcast of an Investment Portfolio:

For withdrawal rates over 3.5%, the sequence of returns is the largest determinant of the success in a retirement portfolio. Yet its effect is missed by all man-made simulators that are based on Gaussian models. That is because of their inherent flaw: They can simulate the volatility of returns rather well. However, they cannot model the patterns of sequence of returns. These patterns are as a result of specific correlations between various economic factors such as equities, bond yields, interest rates and inflation in typical market cycles.

Aftcasting reflects the sequence of returns exactly as it happened in history. Aftcasting, as opposed to forecasting, is a method developed by the author for analyzing investment outcomes. It includes the actual historical equity performance, inflation rate and interest rate, as well as the actual historical sequencing of these data sets.

Aftcasting displays the outcome of all historical asset values of all portfolios since 1900 on the same chart, as if a person starts his plan in each of the years between 1900 and 2000. It gives a bird’s-eye view of all outcomes. It also provides the success and failure statistics with exact historical accuracy, as opposed to man-made simulation models.

Let’s work thorough an example: Bob, 35, has just received \$100,000 inheritance. He wants this money to grow over the next thirty years, in time for his retirement. He wants to know how much money he will have at that time.

His current asset mix is 60% equities and 40% fixed income². The aftcast of this scenario is depicted in Figure 2. On this chart, we see the aftcast lines (thin gray lines) for each starting year since 1900. There are 40 years of data on each aftcast line for all starting years before 1972. After 1971, each aftcast line ends at the end of year 2010. Thus, there are 3706 data points that reflect the exact, actual market history that is exactly in-line with realistic correlations and patterns of performance of equities, bond yields, interest rates and inflation.

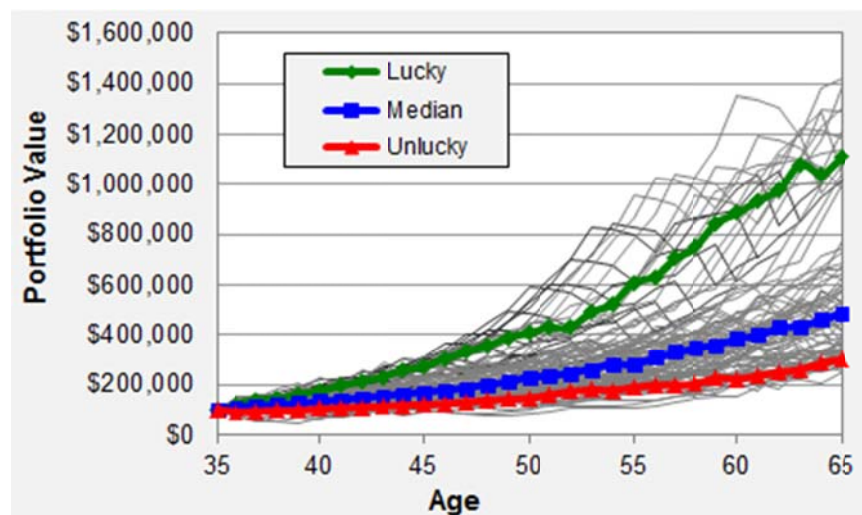
² Equity proxy: Dow Jones Industrial Average since 1900, using currently prevailing dividend rates of 2%, total annual portfolio cost is 2% of the portfolio value, including management fees and all trading costs; a net alpha of 0%. Fixed income net returns (after all expenses), are historic 6-month CD rate plus 0.5%.

We also see “lucky”, “median” and “unlucky” lines. Let’s define what these terms mean:

- The bottom decile (the bottom 10%) of all aftcast lines is the “unlucky” outcome. If you design your plan such that it provides a lifelong retirement income for the unlucky outcome, then that means it only has 10% probability of failure; i.e. you would have a successful outcome³ with 90% certainty.
- The top decile (the top 10%) of all aftcast lines is the “lucky” outcome.
- The blue line indicates the median outcome where half of the scenarios are better and half are worse. If you design a plan that provides a lifelong retirement income for the median outcome, then you would have a successful outcome only with 50% certainty.

We observe that if Bob were lucky during his accumulation stage, his investments would grow to about \$1.1 million, an eleven-fold increase by age 65. On the other hand, if he happened to be unlucky, his \$100,000 initial investment would grow to about \$301,000, about a three-fold increase. And finally, if Bob had median returns, his investments would grow to about \$488,000 by age 65.

Figure 2: The aftcast of an investment portfolio, starting capital of \$100,000



³ Throughout this paper the terms “successful”, “unsuccessful”, “failure”, “certainty” and any similar words refer to statistical outcomes of the market history since 1900. The future outcomes will likely be different.

The Effect of Asset Allocation and Time Horizon:

We used a 60/40 asset mix and a 30-year time horizon to generate the aftcast chart in Figure 2. Let's see what happens if we use different asset mixes and different time horizons. We want to know how much initial capital we need to accumulate \$1 million at the end of that time horizon.

The following table shows the accumulation aftcast for various asset mixes and time horizons:

Table 1: Initial capital required to accumulate \$1 million over a fixed time horizon

Asset Mix		Time Horizon			
Equity/Fixed Income	Outcome	10 years	20 years	30 years	40 years
Initial Capital Required to Accumulate \$1 million:					
100 / 0	Unlucky	\$1,130,785	\$811,877	\$577,341	\$455,221
	Median	\$632,899	\$357,898	\$241,373	\$130,763
	Lucky	\$304,642	\$146,705	\$83,789	\$72,484
80 / 20	Unlucky	\$934,996	\$635,812	\$413,363	\$284,953
	Median	\$603,107	\$339,374	\$209,754	\$127,187
	Lucky	\$343,971	\$161,481	\$85,781	\$64,221
70 / 30	Unlucky	\$857,114	\$597,060	\$367,147	\$252,464
	Median	\$598,940	\$338,973	\$208,719	\$130,801
	Lucky	\$359,505	\$163,557	\$89,301	\$61,711
60 / 40	Unlucky	\$839,198	\$529,279	\$332,041	\$226,836
	Median	\$593,134	\$356,152	\$204,761	\$134,216
	Lucky	\$365,221	\$164,428	\$90,175	\$58,699
50 / 50	Unlucky	\$788,195	\$501,582	\$318,550	\$202,869
	Median	\$587,763	\$342,248	\$212,982	\$144,722
	Lucky	\$381,303	\$177,255	\$91,801	\$56,870
40 / 60	Unlucky	\$776,093	\$498,859	\$319,146	\$201,406
	Median	\$589,574	\$348,833	\$222,976	\$159,853
	Lucky	\$402,443	\$190,111	\$94,571	\$56,049
30 / 70	Unlucky	\$781,157	\$514,591	\$342,380	\$211,926
	Median	\$596,959	\$351,885	\$232,896	\$167,379
	Lucky	\$416,678	\$185,423	\$95,331	\$55,754
20 / 80	Unlucky	\$785,728	\$552,768	\$368,255	\$231,768
	Median	\$596,591	\$350,210	\$227,016	\$170,282
	Lucky	\$435,486	\$179,737	\$97,823	\$55,275
0 / 100	Unlucky	\$853,901	\$678,189	\$497,117	\$319,818
	Median	\$598,130	\$358,171	\$243,223	\$192,972
	Lucky	\$410,559	\$182,303	\$93,929	\$58,461

In many cases, the client has no initial capital to start with, but he plans to save periodically for his retirement. Table 2 indicates the dollar amount that must be saved in the first year of the plan. After the first year, the dollar amount of annual savings is indexed to inflation over the entire time horizon.

Table 2: Annual savings required to accumulate \$1million over a fixed time horizon, starting with no initial savings:

Asset Mix Equity/Fixed Income Outcome		Time Horizon			
		10 years	20 years	30 years	40 years
Annual Savings Required to Accumulate \$1 million: (the first year amount is indicated, in subsequent years, it is indexed to inflation)					
100 / 0	Unlucky	\$95,834	\$40,077	\$20,212	\$10,410
	Median	\$72,827	\$24,342	\$10,083	\$4,916
	Lucky	\$49,528	\$14,293	\$5,080	\$2,561
80 / 20	Unlucky	\$91,114	\$35,029	\$16,971	\$8,918
	Median	\$72,533	\$24,291	\$10,382	\$5,241
	Lucky	\$51,963	\$15,420	\$5,607	\$2,647
70 / 30	Unlucky	\$87,250	\$33,084	\$16,190	\$8,593
	Median	\$72,181	\$24,215	\$10,514	\$5,448
	Lucky	\$54,224	\$15,395	\$5,966	\$2,742
60 / 40	Unlucky	\$84,549	\$31,451	\$15,583	\$8,290
	Median	\$72,389	\$23,689	\$10,827	\$5,604
	Lucky	\$54,925	\$15,612	\$6,173	\$2,857
50 / 50	Unlucky	\$83,723	\$31,569	\$15,270	\$8,121
	Median	\$72,496	\$24,351	\$11,033	\$5,913
	Lucky	\$56,093	\$15,845	\$6,442	\$2,966
40 / 60	Unlucky	\$82,317	\$31,151	\$15,540	\$8,196
	Median	\$72,418	\$24,449	\$11,397	\$6,207
	Lucky	\$57,581	\$16,078	\$6,743	\$3,098
30 / 70	Unlucky	\$82,101	\$31,349	\$15,692	\$8,245
	Median	\$72,009	\$24,948	\$11,708	\$6,218
	Lucky	\$58,661	\$16,380	\$6,629	\$3,105
20 / 80	Unlucky	\$81,706	\$32,453	\$15,814	\$8,674
	Median	\$72,704	\$24,864	\$11,821	\$6,346
	Lucky	\$60,723	\$16,935	\$6,487	\$3,092
0 / 100	Unlucky	\$85,354	\$34,752	\$18,562	\$10,503
	Median	\$72,820	\$25,155	\$11,879	\$6,403
	Lucky	\$61,161	\$16,097	\$6,488	\$3,083

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By using the information on these two tables (Table 1 and Table 2), one can calculate any unknown amount.

Example 1: Jane has \$230,000 in her investment account, 60% equity and 40% fixed income. She saves \$15,000 each year and this dollar amount is indexed to inflation. How much would she accumulate in her account in 20 years with 90% certainty (unlucky outcome)?

Step 1: Initial savings: How much does her \$230,000 grow? We read from Table 1 that at 60/40 asset mix, you need \$529,279 for a \$1 million end value after 20 years if unlucky. Therefore, her existing \$230,000 grows to \$434,553, calculated as \$230,000 divided by \$529,279 multiplied with \$1million.

Step 2: Annual Savings: How much does her annual savings of \$15,000 grow? We read from Table 2 that you need to save \$31,451 in the first year (indexed to inflation in subsequent years) to accumulate \$1 million at the end of 20 years for the unlucky outcome for the 60/40 asset mix. Therefore, her annual savings grows to \$476,932, calculated as \$15,000 divided by \$31,451 multiplied by \$1 million.

Add the results from step 1 and step 2 to calculate the total expected assets. Thus, Jane can expect to have at least \$911,485 with 90% historical certainty in 20 years.

Example 2: Keith has \$350,000 in his investment account, 70/30 asset allocation. How much does he need to save annually for the next 10 years to accumulate \$850,000 with 90% certainty (unlucky)?

Step 1: Initial Savings: How much does his \$350,000 grow? We read from Table 1 that at 70/30 asset mix, you need \$857,114 for a \$1 million end value after 10 years for the unlucky outcome. Therefore, his existing \$350,000 grows to \$408,347, calculated as \$350,000 divided by \$857,114 multiplied with \$1million.

Step 2: Annual Savings: He needs to save sufficient money each year to make up the shortfall of \$441,653, calculated as \$850,000 minus \$408,347. We read from Table 2 that you need to save \$87,250 in the first year (indexed to inflation in subsequent years) to accumulate \$1 million at the end of 10 years for the unlucky outcome for the 70/30 asset mix. Therefore, Keith needs to save annually \$38,534 in the first year (indexed to inflation each year after that), calculated as \$441,653 divided by \$1 million multiplied by \$87,250.

The Most Likely Outcome:

The next question we need to ask is this: Which outcome should we use? Do we use the lucky outcome, the median outcome, or do we use the unlucky outcome?

We need to use the outcome that is most likely to occur for the investor. For that, let's look at the psychology of loss.

The Psychology of Loss:

From the beginning of 1900 until the end of 2009, the compound annual return (CAR) of the DJIA (index only) was 4.7%. However, if you miss the best 39 months out of the entire 1317 months that markets were open for business, your CAR drops down to 0%. This is about 3% of the entire time period. Essentially, what made money for the investor is the extreme good markets that happened only 3% of the time. Conversely, what creates catastrophic losses happened only 3% of the time. These are the “black swan” events that create cataclysmic changes to outcomes where models based on random distributions fail to recognize⁴.

The investor psychology can be compared to how we feel in extreme weather conditions. When it is really cold, the weatherman talks about the wind-chill effect. He might say something like “The temperature is 20 degrees Fahrenheit, but the wind-chill factor makes you feel like minus 10!” On the other hand, when it is really hot, the weatherman might say something like “The temperature is 90 degrees Fahrenheit, but with the heat index⁵ it feels like 120!”

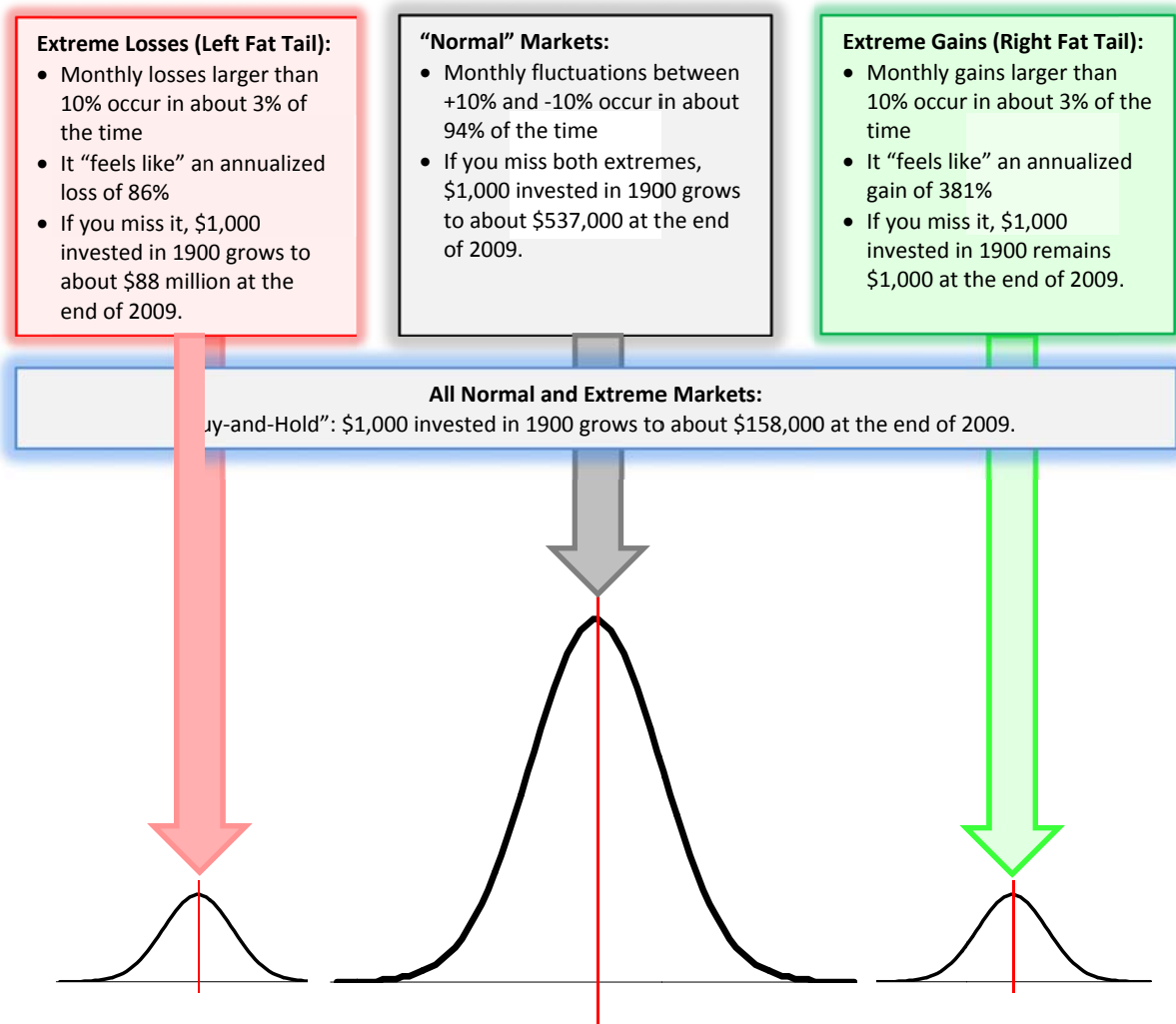
If you look at the extreme bullish trends that occurred during 3% of the time, the annualized growth rate was 381%. During such a time period, after observing this phenomenal increase for a short while, like with the heat index, the investor “feels” like he is missing the boat. Just before that trend turns around, he abandons his long-term asset allocation policies and becomes aggressive with his investments. This action creates conditions for larger future losses.

At the opposite end of the spectrum, during the most extreme bearish trends that occurred 3% of the time, the annualized rate of loss was 86%. During such a time period, after observing this phenomenal loss, like the wind-chill effect, the investor “feels” like he is losing everything. Just before that trend turns around, he abandons his asset allocation policy and sells everything. This action creates conditions for smaller future gains. Figure 3 depicts this in a Gaussian framework.

⁴ The conventional investment wisdom, such as the efficient frontier, MPT, portfolio optimizations, correlation factors between asset classes, and many other concepts are entirely based on the randomness of the market events and are ineffective during these extreme market trends.

⁵ known as “humidex” in Canada

Figure 3: Depiction of Extremes and Normals from the Gaussian vantage point



There are several studies that compare the average market returns versus the investor returns. DALBAR’s 2010 update of its Quantitative Analysis of Investor Behavior (QAIB) study⁶ found that while the S&P 500 has returned 8.35% over a 20 year period ending in 2008, the average equity investor earned just 1.87%. John Bogle⁷ estimates the over a 25 year period ending in 2005, the average mutual fund investor earned 7.3% compared to the 12.3% for the benchmark.

⁶ Dalbar's 2010 Quantitative Analysis of Investor Behavior (QAIB), Dalbar.com

⁷ Bogle John, “The Little Book of Commonsense Investing” ISBN: 978-0470102107

While the financial industry might talk about an “average return” for the market index, the average return for an **average** investor is the **bottom decile** of that market index.

Investor behavior and basic engineering principles compel us to use the bottom decile outcome (the unlucky outcome) as the basis of our calculations. This ensures that a client has a 90% of certainty⁸ of achieving his goals.

Those readers, who have been accustomed to the Gaussian mindset, might think that it is too stringent to use the unlucky outcome for 90% certainty. After all, pension funds assume an *average* growth rate to project 5 years, 10 years, 20 years, even 40 years into the future⁹. My response is; this is exactly why most are failing or on the path towards failure.

In engineering, if you are designing anything, then you have no choice but to design for the adverse conditions and not for the average. Only then, can your design be considered robust and reliable. On the other hand, most forecasts -be it retirement plans for individuals or projections for pension funds- are unable to recognize this “Grand Canyon” between this “*average*” outcomes and “*design*” outcomes.

Optimum Asset Mix:

Next, we need to figure out the optimum asset mix based on aftcasting.

Since the most likely outcome for the investor is the unlucky outcome, and the basic engineering principles dictate that we design for that, we need to figure out the optimum asset allocation based on the unlucky outcome.

We plot the initial capital required for the unlucky outcome for various asset mixes and time horizons. On the graph, we locate the lowest point. This is the lowest initial capital required to yield the same unlucky future value¹⁰. We then read the asset mix that corresponds to this lowest point.

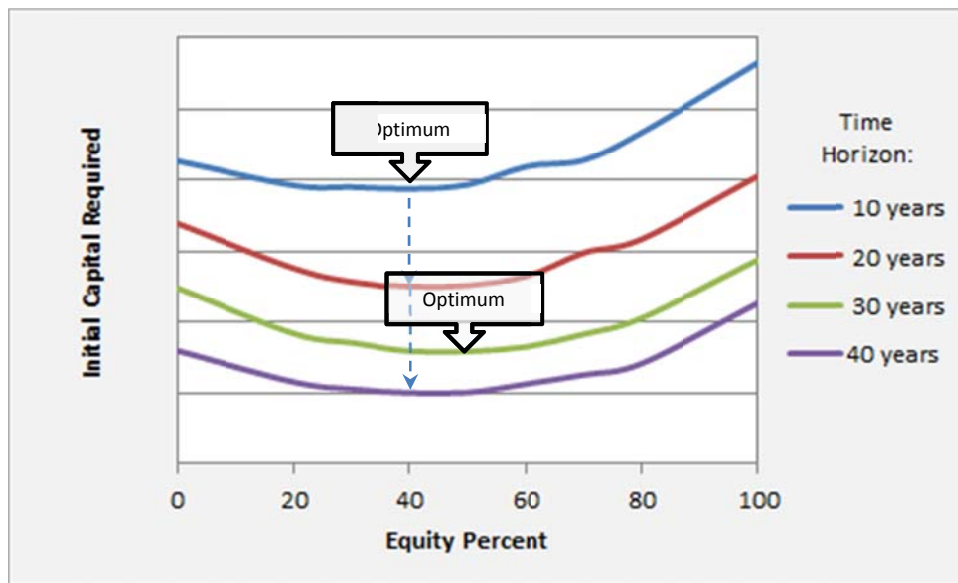
⁸ The term “certainty” is used here in a statistical context. It refers to market history since 1900. Future outcomes will be different.

⁹ Canada Pension Plan projects 75 years into the future

¹⁰ or expressed differently: the maximum future value for the same initial required capital

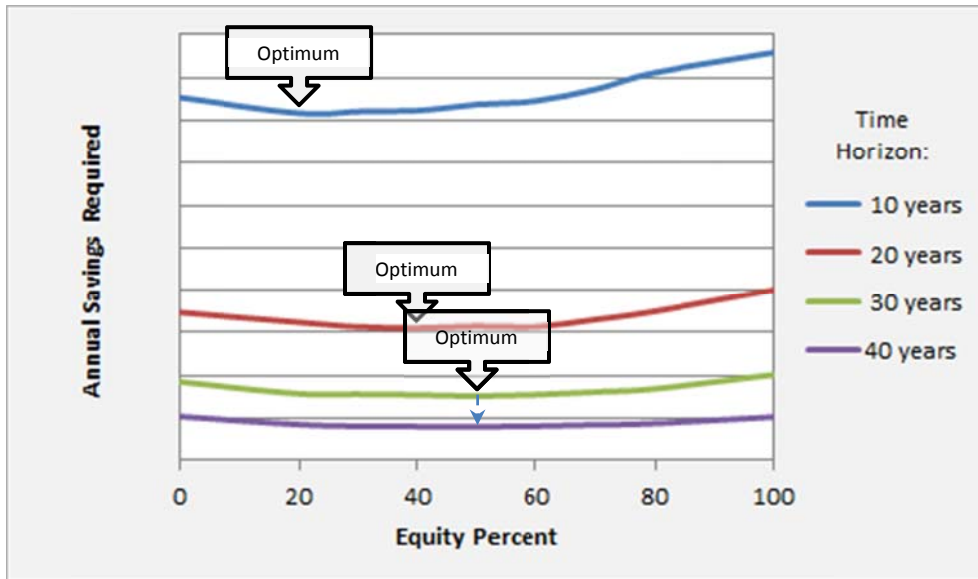
Figure 4 illustrates this. We observe that the lowest point is about 40% equity and 60% fixed income for 10, 20, and 40 year time horizons. This is a far cry from the common wisdom of holding much larger percentage of equities in hopes of higher returns. Yes, you can potentially have higher returns with higher equity content, but that comes at a greater expense of much higher certainty of missing your target.

Figure 4: Determining the optimum asset mix for the unlucky outcome, no annual savings, growth of initial capital only.



If we were to start with no initial assets and only consider dollar-cost-averaging annually (Table 2), then the optimum asset allocation is slightly different. This is depicted in Figure 5. For a 10-year time horizon, the optimum asset mix is 20/80 (20% equity and 80% fixed income). For a 20-year time horizon, it is 40/60. And finally, for 30 and 40-year time horizons, it is 50/50.

Figure 5: Determining the optimum asset mix for the unlucky outcome, no initial capital, annual savings only.



The optimum asset mix for the 90% certainty of achieving your target is summarized in Table 3:

Table 3: Optimum asset mix for 90% certainty (Equity proxy: DJIA):

Time Horizon	Optimum Asset Mix	
	Initial savings only, no annual savings	Annual savings only, no initial savings
10 years	40 / 60	20 / 80
20 years	40 / 60	40 / 60
30 years	50 / 50	50 / 50
40 years	40 / 60	50 / 50

These optimum asset allocation figures indicate that our aftcasting demands a far more conservative portfolio than the conventional asset allocation recommendations using efficient frontier and other similar Gaussian tools. The reason for that is two-fold:

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1. Gaussian methods optimize using the random distribution of market volatility (remember the concept of “standard deviation of returns”). The reality is, in the long term, market might be in a secular bullish trend lasting 20 years (1920-1929, 1949-1962, 1982-1999). It might be in a secular sideways trend lasting as long as 20 years (1900-1920, 1938-1948, 1964-1982). It might also be in a secular bearish trend similar to Nikkei 225 index over the last 25 years. Gaussian tools are insufficient to account for trend discontinuities that have the greatest impact on wealth creation¹¹, wealth preservation, and wealth destruction.

Figure 6: Index value over time (on a semi-log scale) in a randomly generated simulation

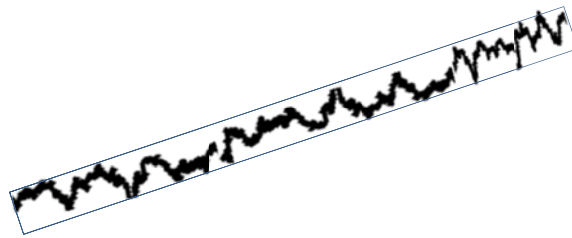
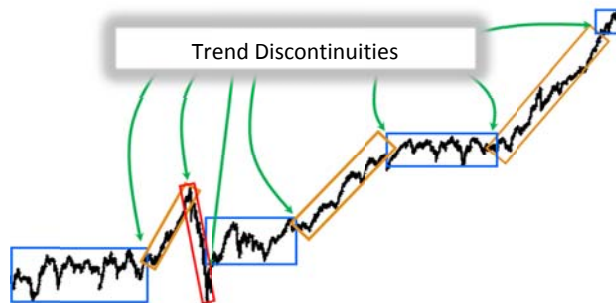


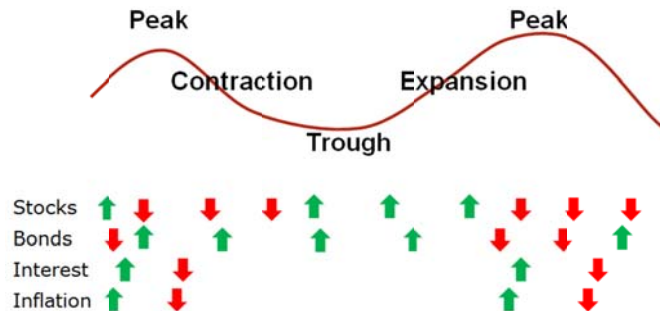
Figure 7: Index value over time (on a semi-log scale) of actual market history starting in 1900



¹¹ Otar, Jim C., Unveiling the Retirement Myth, ISBN: 978-0-9689634-25, Chapter 27 “If You Miss the Best”, page 285

- Each secular trend is made up of cyclical trends which are generally caused by business cycles. The cyclical trends create specific **sequence of events**; which we define as “patterns of directions in inflation, interest rates, bond yields and equities with a specific array of inter-correlations and phase gaps”.

Figure 8: Cyclical trends



There are four distinct phases and four distinct objects in this pattern. When randomized, there is one in sixteen chance of modeling this particular sequence of events correctly. In other words, when you run sixteen thousand random simulations, only one thousand of those will likely have the correct pattern. The remaining fifteen thousand simulations will not only be wrong but they will render the entire outcome useless. The unbounded, fanning-out effect of the accumulation outcomes visible in Monte Carlo simulations do not happen in real life; an extreme in one direction eventually begets another extreme in the opposite direction.

Once you subscribe to the idea of randomness, then you are compelled to accept a much shorter time period as “long term”. Many in the investment industry define a 10-year time horizon as long term. We observe that a sideways secular trend can last twenty years. You need about one and a half times of that to call it long-term. Therefore, we define “long term” as no less than 30 years.

With our definition of “long term”, once the investor is over the age of 40, his/her investment time horizon can no longer be described as long term. Yet, most do not seriously start saving until well after 40. For them, the concept of long term does not really exist.

Some academics define the “retirement risk zone” as the 5-year time periods immediately prior to and following the retirement age. We disagree. We believe that the retirement risk zone starts as soon as the first dollar is invested and it ends when both you and your spouse die. The source and the type of risk changes and shifts over time, but it is always there.

In the current portfolio management practice, efficient frontier analysis is the basis of asset allocation decisions. It is based on standard deviation of returns which is inherently based on Gaussian model. This generally discounts the extremes and therefore their equity allocation is much higher than what we discovered with aftcasting.

In this paper, we use DJIA as our equity proxy. However, we include below (Table 4) the optimum asset mix for S&P500 as equity proxy. And for our Canadian readers, Table 5 displays the same for the SP/TSX index.

Table 4: Optimum asset mix for 90% certainty (Equity proxy: S&P500):

Time Horizon	Optimum Asset Mix	
	Initial savings only, no annual savings	Annual savings only, no initial savings
10 years	30 / 70	30 / 70
20 years	40 / 60	30 / 70
30 years	30 / 70	40 / 60
40 years	40 / 60	30 / 70

Table 5: Optimum asset mix for 90% certainty (Equity proxy: SP/TSX):

Time Horizon	Optimum Asset Mix	
	Initial savings only, no annual savings	Annual savings only, no initial savings
10 years	40 / 60	20 / 80
20 years	40 / 60	70 / 30
30 years	60 / 40	80 / 20
40 years	70 / 30	100 / 0

The optimum asset mix figures we observe using aftcasting are very much in-line with the equity allocation that Benjamin Graham suggested in his book¹² “The Intelligent Investor”, which is described by Warren E. Buffet as “by far the best book on investing ever written”.

¹² Graham, Benjamin, The Intelligent Investor, ISBN: 0-06-015547-7, Chapter 4 “The General Portfolio Policy”

Strategic Asset Allocation - Establishing the Base Case:

Strategic asset allocation means that we establish some “suitable” asset mix for the client and maintain that asset mix over the time horizon. This asset mix is maintained by rebalancing the portfolio periodically, usually annually, sometimes more often. Our base case uses strategic asset allocation for the entire accumulation time horizon.

We will leave out the calculations for the case “starting with nothing and adding only annual savings to a portfolio” from our analysis¹³. Then we end up with a single question to answer: “If I have a fixed amount of dollars to start with, which strategy or product will give me the highest dollar amount at the end of my time horizon with a 90% certainty?”

To keep things simple, we use a 40/60 asset mix as our optimum for all time horizons.

Having determined the optimum asset mix, we now have our base case. We use this base case as the benchmark to measure the capital accumulation efficiency of all other strategies and products.

Table 6: The Base Case for 90% certainty:

Time Horizon	Initial Capital Required to Accumulate \$1 million	Capital Accumulation Efficiency
10 years	\$776,093	100%
20 years	\$498,859	100%
30 years	\$319,146	100%
40 years	\$201,406	100%

Now, we can compare different variables, strategies and products to this base case.

¹³ This paper is not about the optimum asset mix over a life cycle. For that, refer to author’s book “Unveiling the Retirement Myth, Chapter 19

The Effect of Alpha:

Alpha quantifies excess return of equities over and above its benchmark index.

In this paper, we use alpha as a “catch-all” number for the overall equity performance relative to the index for any factor:

- Dividends increase alpha.
- Management fees, portfolio costs, bid/ask spreads, trading costs, taxes generally decrease your alpha.
- Actively managed funds that beat the index over the long term consistently¹⁴, can increase alpha.
- Using technical analysis tools successfully can increase alpha. On the other hand, following one’s emotions for market timing can decrease alpha.

For example, we run an aftcast for an alpha of +2% for an asset mix of 40/60. This includes all effects as described above for the equity portion of the portfolio. We find that you would need \$426,153 as your initial capital for a 20 year time horizon to accumulate \$1 million with 90% certainty.

Remembering that for our base case (see Table 6), we need \$498,859 as initial capital, we calculate the capital accumulation efficiency of 2% alpha over 20 years as following:

$$\text{CAE} = (\$498,859 / \$426,153) \times 100\% = 117.1\%$$

Table 7 summarizes the capital accumulation efficiency for various alpha values and time horizons at the optimum asset mix for 90% certainty.

¹⁴ The author’s experience has been that about 2% to 3% of portfolio managers beat the index consistently over the long term outside the realm of luck. This is about the same proportion of “extreme” versus “normal” markets.

Table 7: The capital accumulation efficiency of various alpha values for 90% certainty:

Alpha	Time Horizon			
	10 years	20 years	30 years	40 years
Capital Accumulation Efficiency:				
-4%	85.2%	72.6%	62.1%	53.5%
-3%	88.7%	78.9%	70.2%	62.7%
-2%	92.3%	85.3%	78.6%	73.1%
-1%	96.1%	92.4%	88.7%	85.8%
0%	100.0%	100.0%	100.0%	100.0%
+1%	104.1%	107.9%	112.4%	116.8%
+2%	108.3%	117.1%	125.7%	136.3%
+3%	113.2%	126.9%	141.1%	159.2%
+4%	117.6%	136.8%	158.0%	185.1%

A reality check: Let's calculate the effective compound annual returns for all alphas and time horizons. Table 8 displays the results.

Table 8: The effective compound annual growth rates of portfolios with various alpha values for 90% certainty of outcome at the optimum asset mix:

Alpha	Time Horizon			
	10 years	20 years	30 years	40 years
Effective Compound Annual Growth Rate:				
-4%	0.9%	1.9%	2.2%	2.5%
-3%	1.3%	2.3%	2.7%	2.9%
-2%	1.8%	2.7%	3.0%	3.3%
-1%	2.2%	3.1%	3.5%	3.7%
0%	2.6%	3.5%	3.9%	4.1%
+1%	3.0%	3.9%	4.3%	4.5%
+2%	3.4%	4.4%	4.7%	4.9%
+3%	3.8%	4.8%	5.1%	5.3%
+4%	4.2%	5.2%	5.5%	5.7%

At the time of writing, the interest rates were near their historically-low levels. A 10-year government bond currently yields about 3.5%. If we allow ourselves to step into a fantasyland where these rates will remain "forever", then we can paint the effective growth rates on Table 6 with a different brush:

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Alpha	Time Horizon			
	10 years	20 years	30 years	40 years
	Effective Compound Annual Growth Rate:			
-4%	0.9%	1.9%	2.2%	2.5%
-3%	1.3%	2.3%	2.7%	2.9%
-2%	1.8%	2.7%	3.0%	3.3%
-1%	2.2%	3.1%	3.5%	3.7%
0%	2.6%	3.5%	3.9%	4.1%
+1%	3.0%	3.9%	4.3%	4.5%
+2%	3.4%	4.4%	4.7%	4.9%
+3%	3.8%	4.8%	5.1%	5.3%
+4%	4.2%	5.2%	5.5%	5.7%

We painted all areas where the effective compound annual return of the portfolio is below the current 10-year bond yield in yellow. Here is our conclusion:

- If your time horizon is 10 years or less *and* you want a 90% certainty of achieving your accumulation target *and* you have no guarantee of beating the index by at least 3%, then you are better off buying guaranteed deposits/investments, such as bonds, FIA or VA-GMWB.
- If your time horizon is 20 years or longer *and* the equity portion of your portfolio is not expected to outperform the index on a consistent basis for the entire time horizon, then you are better off holding broad-based, low-cost ETFs in the equity portion of your portfolio.
- Disregard any performance numbers, including any greek letters, for any actively managed portfolios or funds (regardless of holding individual stocks or trading ETFs) with a history under 20 years. They do not indicate anything except presence or absence of luck.
- If your time horizon is 30 years or longer *and* you expect to beat the index, then you can try to seek active managers with a good track record, hoping and monitoring that they continue their performance.
- “Normal Yield” Scenario: If bond yields eventually increase towards their historical averages, then the painted area on this table might move even lower. In that case, the entire 10-year time horizon, as well as most of the 20-year time horizon, can become a too short of a time horizon to invest for the 90% certainty of outcome that you require.
- “Extreme Yield” Scenario: If bond yields eventually increase far above their historical averages –perhaps indicating increased default risk of the bond issuer-, then there is not much you can do, most portfolios will lose.

The Effect of Fixed Income Yields:

In our optimum asset mix, we assumed in the fixed income portion of our portfolio a yield that is at a 0.5% premium over and above the historical, annualized interest of a 6-month CD. This number matches approximately the current bond yields.

We varied this premium to determine the capital efficiency for different bond yield premiums to observe their sensitivity. Table 9 summarizes the results.

Table 9: The capital accumulation efficiency for various average yields of the fixed income portion of the portfolio for 90% certainty:

Bond Yield Premium over and above the historical, annualized 6-month CD interest	Time Horizon			
	10 years	20 years	30 years	40 years
	Capital Accumulation Efficiency:			
0.0%	97.1%	94.1%	91.5%	88.6%
0.5%	100.0%	100.0%	100.0%	100.0%
1.0%	102.9%	105.9%	108.9%	112.1%
1.5%	106.0%	112.2%	118.5%	126.1%
2.0%	109.0%	118.8%	129.0%	140.4%
2.5%	112.2%	125.9%	140.3%	157.8%

These figures indicate that for accumulation portfolios, the performance of the bonds is just as important as the performance of equities, and especially so over the longer term. Keep in mind, extreme low bond yield premiums that the North American markets is experiencing during the last few years, can go on for long periods of time, as is the case in the Japanese markets for the last couple of decades.

The Effect of Inflation Indexed Bonds:

So far, we used conventional bonds in the fixed income portion of our portfolios. Now, we analyze inflation indexed bonds as our fixed income.

Unlike conventional bonds, the coupon payments of inflation indexed bonds are indexed to inflation in some form. That protects the bondholder against inflation.

The inflation indexed bonds do not have too long of a history. They have become popular during the last 20 years. Initial buyers were mostly pension funds and insurance companies which needed to mitigate their inflation-linked liabilities. Because there is insufficient history, we aftcast various levels of real returns of inflation indexed bonds by using the historical CPI plus a real yield over inflation. However, before rushing to present you with tables, we would like to share our findings about the variability of our assumptions first. It is important to understand why using one “average” real return will not be sufficient for a robust analysis.

Let’s look at secular trends to see the interrelation between inflation and market growth. Table 10 indicates the average annual inflation and DJIA growth over each secular trend.

Table 10: Inflation in secular trends (1900 – 1999)

	Trend	Average Annual DJIA Growth	Average Annual Inflation	Length, years
All Trends	1900 – 1999	7.7%	3.3%	
Secular Sideways¹⁵:		2.4%	5.6%	
	1900 – 1920	4.2%	4.8%	21
	1937 – 1948	1.4%	4.8%	12
	1966 – 1981	0.8%	7.1%	16
Secular Bull:		15.0%	1.8%	
	1921 – 1928	20.6%	-1.5%	8
	1949 – 1965	11.5%	1.7%	17
	1982 – 1999	15.9%	3.3%	18
Secular Bear:		-31.7%	-6.4%	
	1929 – 1932	-31.7%	-6.4%	4
Other:				
Cyclical Bull	1933 – 1936	33.5%	1.7%	4

¹⁵ weighted average; weighted by the length of each secular trend

We observe that:

- During the last century, markets spent about 46% of their time in secular bullish trends. Within these trends, the average annual inflation was 1.8% and the annual average growth of equities was 15% (index only), which handily beats the inflation.
- During the last century, markets spent about half of their time in secular sideways trends. Within these trends, the average annual inflation was 5.6%, which is much higher than in secular bullish trends. This higher inflation rate was not steady over the entire duration of the secular sideways trend; it generally stayed low during the first two-thirds of the sideways trend and then increased steeply. The annual average growth in equities was only 2.4% (index only), far short of inflation. In secular sideways trends, equities did not provide an inflation hedge.

When holding inflation indexed bonds, you can expect a higher real yield during secular bullish trends. That is because the alternative, equities, provide a much higher return, and during such times and, as time goes on, they are perceived to be less riskier than they actually are. On the other hand, during sideways trends, equities are perceived to be more risky. Thus, inflation indexed bonds can be more attractive and command a relative higher price (lower real interest).

Because of this potential compression/expansion cycle of real returns in different types of secular trends, one should not just pick and choose an “average” real return but a range of averages to cover all situations. This is also true not only for individuals, but also for forecasts concocted for pension plans using the historical “averages” to make forecasts for the next 5 to 75 years. Their picture becomes much different (dimmer) as soon as we apply a specific target with a 90% certainty.

Now, we can continue with our efficiency figures, summarized on Table 11.

Table 11: The capital accumulation efficiency of holding inflation indexed bonds (IIB) as fixed income for 90% certainty:

Asset Mix Equity/IIB	Real Yield over and above the historical CPI	Time Horizon			
		10 years	20 years	30 years	40 years
Capital Accumulation Efficiency:					
80 / 20	0%	79.1%	70.5%	66.1%	57.7%
	1%	80.7%	73.0%	68.4%	61.4%
	2%	82.3%	75.9%	72.3%	65.7%
	3%	83.9%	79.3%	77.6%	72.4%
60 / 40	0%	82.7%	71.5%	68.9%	63.6%
	1%	86.1%	77.3%	78.0%	74.9%
	2%	89.6%	84.1%	88.2%	88.2%
	3%	93.2%	90.9%	100.3%	104.8%
40 / 60	0%	84.2%	69.7%	69.3%	63.6%
	1%	89.8%	78.4%	82.6%	80.9%
	2%	95.8%	88.6%	99.0%	103.1%
	3%	102.1%	99.5%	119.3%	130.3%
20 / 80	0%	84.2%	66.7%	63.9%	54.0%
	1%	90.2%	77.8%	79.7%	73.2%
	2%	97.4%	90.1%	100.6%	99.2%
	3%	105.1%	105.0%	126.6%	134.3%

For reasons described earlier about the compression/expansion of real yields spanning over different secular trends, the appearance of higher CAE in some cases (in Table 11) is not realistic for time horizons over 10 years.

We can conclude that, for accumulation portfolios, unless there is an immediate need for inflation-adjusted periodic withdrawals, adding inflation indexed bonds can create a lower certainty of achieving a specific target than the base case.

Rebalancing Strategies:

There are many rebalancing strategies. If we ignore strategies bordering on market timing, we end up with two: optimizing the level of the threshold and the frequency.

The rebalancing threshold determines how much of a deviation from the optimum asset mix do you need to trigger rebalancing. We look at three different thresholds: 0%, 3% and 6% deviation from the equity percentage in the optimum asset mix.

The rebalancing frequency determines how often we rebalance. We look at three different rebalancing frequencies:

1. Annual rebalancing
2. Rebalance every four years at the end of the Presidential election year to synchronize with the President election cycle of the markets
3. Starting with the optimum asset mix and never rebalance again until the end of the accumulation time horizon

Table 12 summarizes the capital accumulation efficiency for different time horizons with 90% certainty of achieving the target amount.

Table 12: The capital accumulation efficiency for various rebalancing strategies with 90% certainty:

Rebalancing Strategy Frequency / Threshold	Time Horizon			
	10 years	20 years	30 years	40 years
Capital Accumulation Efficiency:				
Annual / 0%	99.5%	99.5%	98.7%	98.6%
Annual / 3%	100.0%	100.0%	100.0%	100.0%
Annual / 6%	99.9%	98.9%	100.3%	100.3%
Pres. Election Year / 0%	103.4%	110.6%	118.7%	111.8%
Pres. Election Year / 3%	103.4%	111.7%	119.3%	113.2%
Pres. Election Year / 6%	103.4%	112.9%	120.0%	112.6%
Never Rebalance	94.9%	84.3%	78.4%	72.9%

During the accumulation stage, rebalancing once every four years at the end of the Presidential election year appears to give the best results¹⁶. This is in contrast with the “the more, the better” approach of the current wisdom about rebalancing.

¹⁶ Otar, Jim C., Unveiling the Retirement Myth, ISBN: 978-0-9689634-25, Chapter 6 “Rebalancing”, page 72

Age-Based Asset Allocation:

Some investors like to reduce their equity exposure as they get older. Many followers of this idea use this simple formula: The equity percentage in their portfolio equals to 100 less their age. Here are equity allocations for selected ages:

Age	Equity Allocation
30	70%
40	60%
60	40%
80	20%

Since the time horizon is loosely a function of investors current age, we need to calculate the capital accumulation efficiency for various retirement ages. Table 13 summarizes the capital accumulation efficiency using age-based asset allocation for different time horizons with a 90% certainty of achieving the target amount.

Table 13: The capital accumulation efficiency of age based asset allocation with 90% certainty:

Accumulation ends at age:	Time Horizon			
	10 years	20 years	30 years	40 years
Capital Accumulation Efficiency:				
60	100.0%	100.0%	89.6%	82.0%
65	101.0%	100.2%	91.4%	82.5%
70	99.9%	98.9%	92.7%	84.3%
75	99.5%	96.7%	92.9%	85.6%

If you are investing for the long term, then using the age-based asset allocation reduces the probability of reaching your target objective compared to our base case for the same starting capital. And the longer the time horizon, the lower is that probability.

Accelerated Age-Based Asset Allocation:

This is a variation of the age-based asset allocation. The formula for calculating the equity exposure is:

$$\text{Equity Percentage} = 100 - \frac{\text{Age}^2}{100}$$

Here are equity allocations for selected ages:

Age	Equity Allocation
30	91%
40	84%
60	64%
80	36%

Table 14 summarizes the capital accumulation efficiency using accelerated age-based asset allocation for different time horizons with 90% certainty of achieving the target amount.

Table 14: The capital accumulation efficiency of accelerated age based asset allocation with 90% certainty:

Accumulation ends at age:	Time Horizon			
	10 years	20 years	30 years	40 years
Capital Accumulation Efficiency:				
60	89.4%	80.1%	71.1%	59.5%
65	91.8%	84.9%	76.3%	61.9%
70	94.4%	89.8%	79.6%	66.9%
75	97.8%	97.0%	83.4%	72.3%

If you are investing for the long term, then using the accelerated age-based asset allocation reduces the probability of reaching your target objective compared to the base case for the same starting capital. The longer the time horizon, the lower is the probability of reaching that target; much more so than the regular (non-accelerated) age based asset allocation.

Target-Date Funds:

Target date asset allocation is a glorified version of the age based asset allocation. The investor selects a certain target date fund such as “XYZ-2040 Target Date Fund”. This fund starts with an aggressive equity holding, but as time gets closer to the target date (usually in 5 or 10-year bands), the equity allocation is reduced. This is commonly called a “glidepath”.

We have seen earlier that both types of age-based asset allocation strategies reduce the equity exposure as you get older. And both have a larger probability of **not** meeting your target accumulation when compared to the base case. Since target-date asset allocation strategies follow a similar pattern to age-based strategy, why would you expect a better outcome?

There are many different target date funds in the marketplace with various levels/slopes of glidepaths. We categorized them into three different levels: Extremely aggressive, Very aggressive, and Aggressive.

We included the adjective “aggressive” in all our category descriptions. This is because we believe that the currently-available glidepaths generally represent a casino-inspired betting and not target-driven accumulation strategies. Table 15 summarizes the glidepaths we used in our analysis.

Table 15: Typical glidepaths in this analysis

Category	Target Time Horizon	Equity Allocation
Extremely Aggressive	30 years and longer	100%
	20 years	85%
	10 years	70%
Very Aggressive	30 years and longer	90%
	20 years	75%
	10 years	60%
Aggressive	30 years and longer	80%
	20 years	65%
	10 years	40%

Table 16 summarizes the capital accumulation efficiency of various target-date asset allocation strategies for different time horizons with 90% certainty of achieving the target.

Table 16: The capital accumulation efficiency of various target date asset allocations with 90% certainty:

Glidepath Category	Time Horizon			
	10 years	20 years	30 years	40 years
	Capital Accumulation Efficiency:			
Extremely Aggressive	90.0%	75.0%	64.4%	51.3%
Very Aggressive	92.8%	84.1%	71.6%	60.7%
Aggressive	99.5%	88.4%	81.1%	68.2%

Target-date funds with similar glidepaths have a **significantly** greater probability of **not** meeting an accumulation target when compared to our base case with a fixed (non-gliding) optimum asset mix. This is true for all time horizons. And, what is worse is, the longer the time horizon, the lower is the probability of reaching that target.

We did not account for the generally higher portfolio costs of the target date funds in this calculation. Therefore, the numbers should be actually a little worse than displayed on Table 16 for many popular target date funds.

Most target date funds -in their current design- will likely be a waste of your precious capital, time and hope. Unfortunately, it will likely take a long time before many investors realize that.

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Tactical Asset Allocation:

Here, the tactical asset allocation refers to the concept of “reversion to mean”. If the equity index growth rate in the most recent calendar year exceeds its average growth rate, then we expect a reversion to that average. That means we reduce our equity holdings to a more defensive asset mix. Conversely, if the equity index growth rate in the most recent calendar year lags its average growth rate, then we increase our equity holdings to a more aggressive asset mix, again, expecting a reversion to mean.

We found¹⁷ that a 6-year look-back is the optimum length for the definition of “mean” or average. It is long enough to cover a typical market cycle, but short enough not to miss the longer secular trend.

Table 17 summarizes the capital accumulation efficiency using tactical asset allocation strategy with various aggressive/defensive combinations for different time horizons with 90% certainty of achieving the target amount.

Table 17: The capital accumulation efficiency using various tactical asset allocations with 90% certainty:

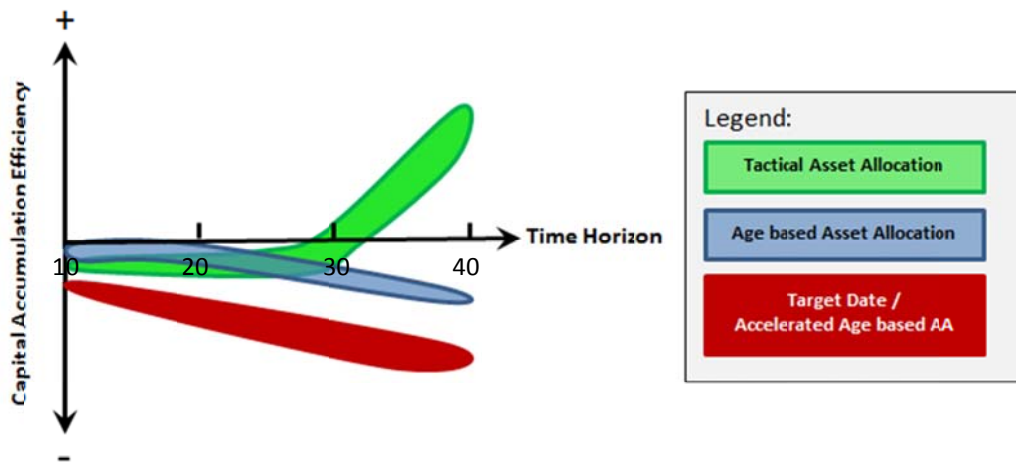
Equity Allocation Aggressive / Defensive	Time Horizon			
	10 years	20 years	30 years	40 years
Capital Accumulation Efficiency:				
100 / 0	84.7%	82.2%	99.0%	163.1%
100 / 20	91.5%	95.4%	102.7%	159.7%
80 / 0	88.1%	88.3%	99.8%	154.9%
80 / 20	97.0%	96.2%	104.7%	152.3%
60 / 0	92.9%	94.3%	97.7%	135.9%
60 / 20	96.1%	99.4%	103.2%	133.6%

This particular tactical asset allocation strategy generally worked better than the base case for time horizons 30 years or longer.

¹⁷ Otar, Jim C., Unveiling the Retirement Myth, ISBN: 978-0-9689634-25, Chapter 24 “Tactical Asset Allocation”

We can summarize the relative attractiveness of various strategies that we looked at. Figure 9 depicts qualitatively the capital accumulation efficiency of various strategies.

Figure 9: Qualitative depiction of capital accumulation efficiency for different strategies



:

Fixed Index Annuities (FIA):

Fixed index annuities¹⁸ guarantee a fixed, minimum interest rate for the term of the contract, usually 10 years. In addition, this interest rate can be higher based on an index's gain (such as the S&P 500). There are several dimensions of how they work and how the interest is credited. With all FIAs, the principal is protected from market risk.

Interest Credit: Two of the most popular methods of earning additional interest are:

1. A participation rate of the underlying index (typically between 50% and 100% of the index growth)
2. A yield spread of underlying index (for example: index growth less 3%)

We used both methods in our analysis.

Interest Crediting Method: There are numerous ways of crediting the interest. We included two of the most popular methods in our analysis:

1. Term End Point: The index movements are measured for the entire term of the FIA and added at the end of the term.
2. Annual Point-to-Point: The index movements are measured and interest is credited and compounded annually.

Caps: A cap is a ceiling on the interest that can be credited. It can be of the type that limits the index cap (the ceiling for the index movement) or interest cap (the ceiling for the interest credit). We used several different cap values.

Minimum Interest: In our calculations, we used 3% minimum annual interest rate on 90% of the premium received.

Underlying Index: We used DJIA as the underlying index.

Term: We used 10-year term for all FIAs. That means for a 30-year time horizon, the FIA was purchased once and then renewed twice for the same product with identical properties.

Similar Products: Index-Linked CD (a.k.a. Index-Linked GIC in Canada) is offered by some banks. We included 5-year-term index-linked CDs in our analysis, renewed for the specified time horizon.

¹⁸ previously called "equity-indexed annuities"

Going back to our earlier example, Bob hears about fixed index annuities from a friend. He wants to know if that is a good way of growing this money.

Bob buys a FIA with the following properties:

- Term: 10 years, annual point-to-point
- Participation rate: 100%
- Annual index cap: 10%
- Minimum guaranteed interest: 3% annual on 90% of the premium

Bob needs to invest his money for 30 years, so after purchasing this FIA, he renews it twice, each for a ten-year term.

Figure 10 depicts the aftcast. We have seen earlier that with the investment portfolio, if unlucky, Bob would have had about \$301,000 at age 65 (base case). With FIA, if unlucky, he would have about \$390,711, which is about 30% higher.

Let's take a small diversion. In this paper, we are not looking at median outcomes. However, because the FIA is much more tinker-proof than an investment portfolio, it is more likely that one would receive the median income from a FIA than the unlucky outcome. If Bob had median returns from the FIA investment, his portfolio would grow to \$506,761, which is about 68% better than the unlucky outcome of the base case. Even though we are not looking at median portfolios in this paper, keep this in mind when reading the capital accumulation efficiency figures on Table 18.

Figure 10: The aftcast of Bob's FIA, starting capital of \$100,000

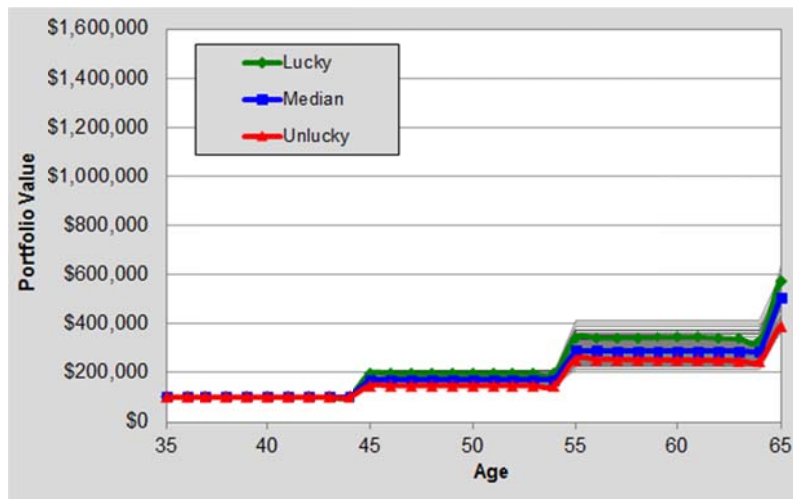


Table 18: The capital accumulation efficiency for various Fixed Index Annuities with 90% certainty:

Description	Time Horizon			
	10 years	20 years	30 years	40 years
Annual Point-to-Point	Capital Accumulation Efficiency:			
50% Participation, no cap	116.5%	127.1%	141.6%	151.7%
60% Participation, no cap	125.7%	151.6%	187.4%	218.8%
70% participation, no cap	135.4%	180.2%	246.9%	316.9%
80% participation, no cap	145.7%	213.6%	323.9%	455.8%
80% participation, 8% interest cap	105.6%	106.1%	96.1%	94.4%
80% participation, 10% interest cap	112.5%	123.0%	120.0%	128.7%
80% participation, 12% interest cap	118.4%	138.8%	146.5%	167.4%
80% participation, 8% index cap	104.3%	92.8%	83.9%	75.7%
80% participation, 10% index cap	105.6%	106.1%	96.1%	94.4%
80% participation, 12% index cap	111.3%	119.8%	114.8%	121.3%
100% participation, 8% interest cap	107.3%	107.7%	98.2%	97.7%
100% participation, 10% interest cap	113.6%	127.0%	124.7%	136.0%
100% participation, 12% interest cap	121.3%	147.4%	155.5%	185.6%
100% participation, 8% index cap	107.3%	107.7%	98.2%	97.7%
100% participation, 10% index cap	113.6%	127.0%	124.7%	136.0%
100% participation, 12% index cap	121.3%	147.4%	155.5%	185.6%
1% yield spread, 8% interest cap	105.6%	106.3%	95.8%	94.4%
1% yield spread, 10% interest cap	112.7%	124.2%	120.9%	130.7%
1% yield spread, 12% interest cap	119.0%	140.9%	149.8%	175.6%
1% yield spread, 8% index cap	104.3%	97.7%	87.1%	82.1%
1% yield spread, 10% index cap	109.6%	115.5%	107.9%	111.3%
1% yield spread, 12% index cap	115.8%	132.3%	134.4%	152.4%
1% yield spread, no cap	159.3%	265.2%	461.9%	737.1%
2% yield spread, 8% interest cap	104.7%	103.3%	94.8%	92.6%
2% yield spread, 10% interest cap	110.6%	118.4%	117.6%	125.8%
2% yield spread, 12% interest cap	116.0%	134.4%	144.3%	165.8%
2% yield spread, 8% index cap	104.3%	90.1%	81.8%	70.8%
2% yield spread, 10% index cap	104.7%	103.3%	94.8%	92.6%
2% yield spread, 12% index cap	110.6%	118.4%	117.6%	125.8%
2% yield spread, no cap	151.0%	236.3%	387.3%	585.9%
4% yield spread, 8% interest cap	104.3%	98.4%	92.7%	87.9%
4% yield spread, 10% interest cap	106.6%	110.6%	110.4%	113.0%
4% yield spread, 12% interest cap	111.7%	124.1%	133.6%	144.1%
4% yield spread, 8% index cap	104.3%	90.1%	77.5%	65.7%
4% yield spread, 10% index cap	104.3%	90.1%	80.5%	69.4%
4% yield spread, 12% index cap	104.3%	98.4%	92.7%	87.9%
4% yield spread, no cap	133.6%	190.4%	276.1%	381.4%

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Description	Time Horizon			
	10 years	20 years	30 years	40 years
Term End-Point	Capital Accumulation Efficiency:			
Participation between 50% and 100%, no cap	104.3%	90.1%	97.3%	113.7%
Participation between 50% and 100%, 100% cap	104.3%	90.1%	97.3%	97.8%
Participation between 50% and 100%, 200% cap	104.3%	90.1%	97.3%	113.7%
Yield Spread between 1% and 4%, no cap	104.3%	90.1%	97.3%	113.7%
Yield Spread between 1% and 4%, 100% cap	104.3%	90.1%	97.3%	97.8%
Yield Spread between 1% and 4%, 200% cap	104.3%	90.1%	97.3%	113.7%
Index-Linked CD, 5-year term, 0% minimum interest				
50% to 70% Participation, no cap	89.2%	96.0%	130.0%	155.2%

Table 18 indicates that FIAs with annual point-to-point interest credit feature can be a great asset class for the purpose of wealth accumulation. If certainty of outcome is important to the investor, having a minimum interest and not losing the principal, makes a huge difference in achieving one's objectives. There are also tax benefits one should not ignore.

However, if the interest crediting method is based on term-end point, then the outcome is not as attractive. That is because in the unlucky case, the FIA accumulates only the minimum interest (in this case; 3% interest on 90% of the premium). Keep this in mind when selecting a FIA for accumulation purposes.

Variable Annuities with Guaranteed Minimum Withdrawal Benefits:

Variable annuities with guaranteed minimum withdrawal benefit (VAGMWB) can be an effective accumulation tool for the purposes of retirement income.

With VA-GMWBL, the income stream for the retiree is paid from his portfolio until it depletes. After that, payments continue seamlessly, but now, it is paid by the insurer for life. The plan may include the retiree's life only or it may include a both spouses.

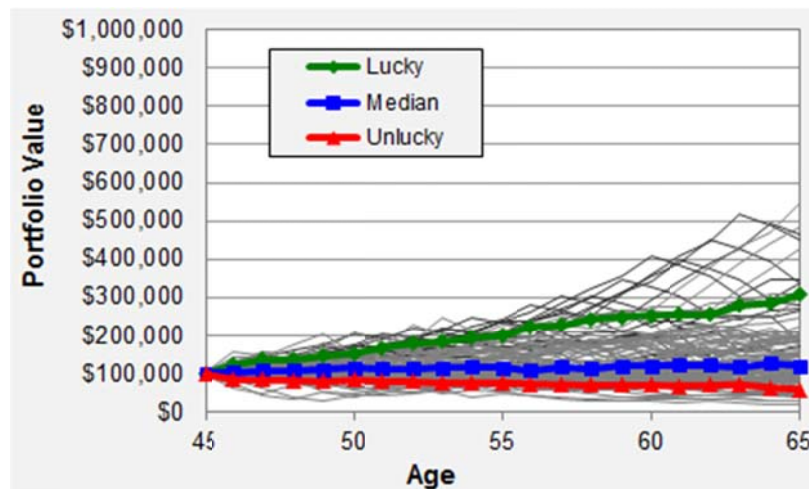
A VA-GMWBL has two balances to keep track of: The first one is the market value, which fluctuates just like any investment portfolio. This is called the Contract Value (CV). The second balance to keep track of is the GWB. It is used to calculate the income payments, which are a percentage of the GWB.

Let's look at an example: Keith, 45, has \$100,000. He needs retirement income in 20 years. He is considering a VA-GMWBL. The VA-GMWBL has the following features:

- The asset mix is 60% equity, 40% fixed income.
- M&E is 2.5% of the portfolio value, the GMWB rider is 1.2% of the GWB.
- Step-ups are reviewed annually. If the market value is higher, then the GWB is stepped-up too. If not, a 5% minimum interest applies (compounded). This minimum interest applies for the first 15 years only.

Figure 11 depicts the aftcast of the market value of Keith's VA.

Figure 11: The aftcast of Keith's VA-GMWBL market value, starting capital of \$100,000

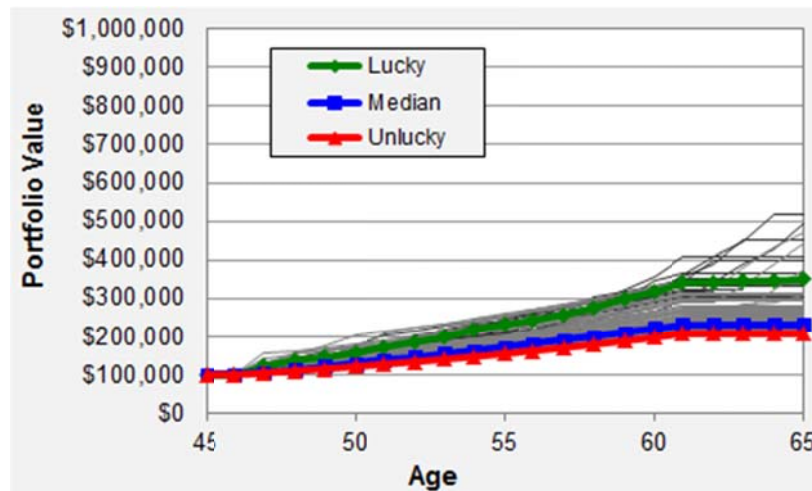


If Keith is unlucky, he would end up with \$58,974; if lucky, with \$307,238; and median, \$120,019 as the market value of his portfolio at age 65.

However, the market value is irrelevant. The income from a VA-GMWB is based on the guaranteed withdrawal base. For this benefit, Keith paid an annual rider of 1.2% for twenty years. Therefore, we aftcast guaranteed withdrawal base balance and not the market value.

Figure 12 depicts the aftcast of the guaranteed withdrawal base of Keith’s VA-GMWB

Figure 12: The aftcast of Keith’s VA-GMWB guaranteed withdrawal base value, starting capital of \$100,000



Now, we see –on the basis of retirement income–, if Keith is unlucky, he would end up with \$207,893; if lucky, with \$348,876; and median, \$231,173 as the guaranteed withdrawal base. This might appear as only slightly better than the unlucky outcome of \$200,457 of the base case (calculated from Table 6 for a 20-year time horizon). However unlike the investment portfolio, with the VA-GMWB, the retirement income is guaranteed for life, even after the portfolio depletes during retirement.

The majority of VA-GMWB products offered in the market provides attractive accumulation features mostly for the first ten to fifteen years of the plan. Therefore, we will only analyze at 10-year and 20-year accumulation time horizons. Table 19 displays the capital accumulation efficiency of various VA-GMWBs.

In all cases listed below, a higher market value can trigger a step-up of the guaranteed withdrawal base for the entire time horizon. Also, the interest and the market value step-ups are stackable, except where noted.

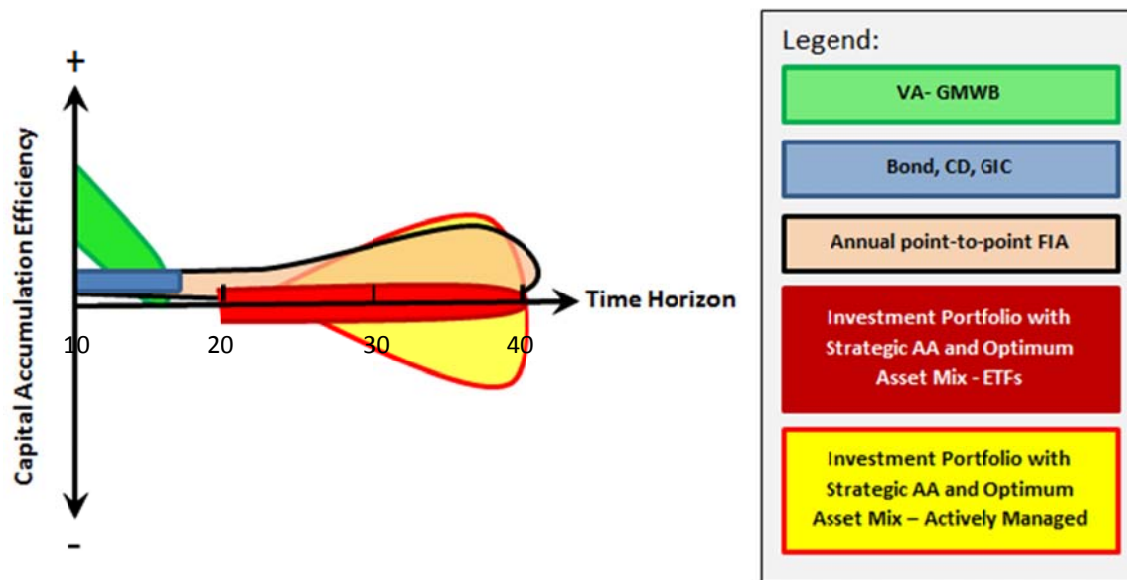
Table 19: The capital accumulation efficiency for various Variable Annuities with GMWB with 90% certainty:

Description	Time Horizon	
	10 years	20 years
	Capital Accumulation Efficiency:	
5% simple, for 10 years,	116.4%	74.8%
5% simple, for 15 years	116.4%	87.3%
5% simple, for 20 years	116.4%	99.8%
6% simple, for 10 years	124.2%	79.8%
6% simple, for 15 years	124.2%	94.8%
6% simple, for 20 years	124.2%	109.7%
8% simple, for 10 years	139.7%	89.8%
8% simple, for 15 years	139.7%	109.7%
8% simple, for 20 years	139.7%	129.7%
10% simple, for 10 years, step-up is non-stackable	155.2%	99.8%
5% compound, for 10 years	126.4%	81.3%
5% compound, for 15 years	126.4%	103.7%
5% compound, for 20 years	126.4%	132.4%
6% compound, for 10 years	139.0%	89.3%
6% compound, for 15 years	139.0%	119.6%
6% compound, for 20 years	139.0%	160.0%
8% compound, for 10 years	167.6%	107.7%
8% compound, for 15 years	167.6%	158.2%
8% compound, for 20 years	167.6%	232.5%
After 10 years, the GWB is set to minimum 200% of starting amount:		
5% simple, for 10 years	155.2%	99.8%
5% simple, for 15 years	155.2%	112.2%
5% simple, for 20 years	155.2%	124.7%
6% simple, for 10 years	155.2%	99.8%
6% simple, for 15 years	155.2%	114.7%
6% simple, for 20 years	155.2%	129.7%
8% simple, for 10 years	155.2%	99.8%
8% simple, for 15 years	155.2%	119.7%
8% simple, for 20 years	155.2%	139.7%

If you are accumulating for retirement income and have a relatively short time horizon (10 to 15 years), a VA-GMWB has significantly higher capital accumulation efficiency than the base case.

Figure 13 depicts qualitatively the capital accumulation efficiency of various product classes.

Figure 13: Qualitative depiction of capital accumulation efficiency for different product classes



Conclusion:

If your purpose is to accumulate **a specific target dollar amount at a given future target date**, then the non-Gaussian aftcast of all years since 1900 shows that:

- The optimum asset mix between equities and bonds points to a much lower equity content in a non-Gaussian approach compared to the Gaussian approach (efficient frontier, MPT).
- The performance of bonds is just as important as the performance of equities in an accumulation portfolio.
- A target-date fund is much more likely to miss a target than a portfolio with a constant optimum asset mix.
- Fixed index annuities, which are of the annual-point-to-point type, have a significantly better chance of achieving a specific future target dollar amount than a portfolio with a fixed, optimum asset mix.
- If the purpose of accumulation is to provide retirement income within the next 10 to 15 years, variable annuities with step-up features also have a significantly higher chance of achieving a specific future target dollar amount.

The capital accumulation efficiency varies with different time horizons. Different products and strategies excel in specific time horizons:

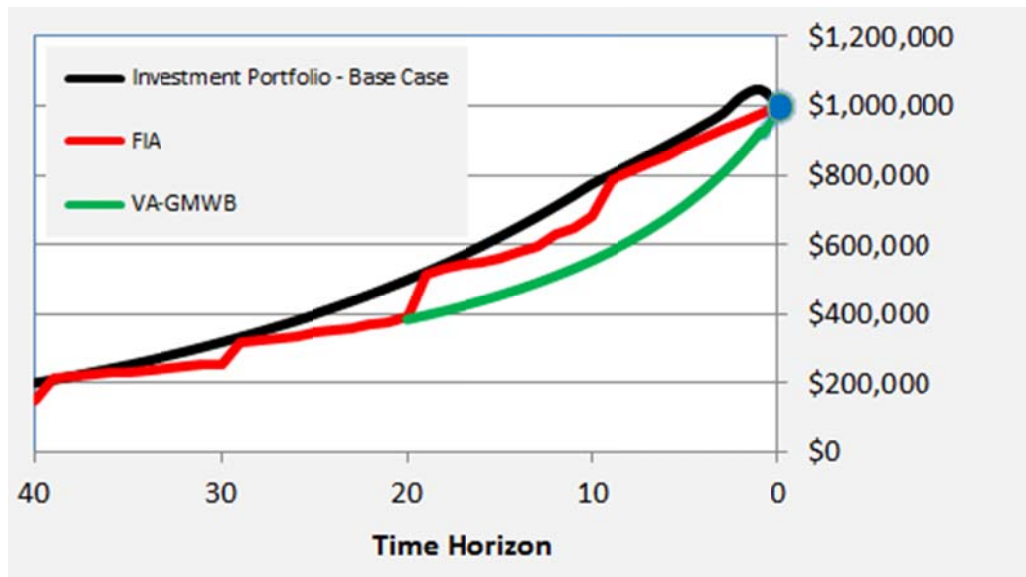
- **Time horizons shorter than 15 years:** Variable annuities with guarantees (VA-GMWB) generally provide the most efficient structure for accumulation of retirement income.
- **Time horizons 20 years and longer:** There are a number of choices:
 1. If you want an absolutely passive instrument and want to minimize the downside risk, then carefully-selected fixed index annuities can work very efficiently for you.
 2. If you don't want to be too passive, you can have a buy-and-hold portfolio with optimum asset mix, holding low-cost, broad based ETFs. You can potentially add to performance with optimum rebalancing or tactical strategies.
- **Time horizons 30 years longer:** If, and only if, you have the capability of selecting excellent managers, you can try a buy-and-hold portfolio with an optimum asset mix and with active management. You can potentially add to performance with optimum rebalancing or tactical strategies. However, once you decide to be active, it is a full-time undertaking; any lapse of monitoring can backfire even with the best of fund managers.

In all situations, you need to do your research on the features of each product and make sure its aftcast works for you for your specific time horizon.

We can combine the asset paths (for 90% of certainty of outcome) into a single chart. Figure 14 depicts the asset paths for three different accumulation vehicles:

1. The black line is the base case, the investment portfolio with a 40/60 asset mix¹⁹.
2. The red line is a FIA. It is of annual-point-to-point type, minimum guaranteed interest rate is 3% on 90% of the premium, 100% participation, 10% interest cap, 10-year term, any time period less than the term collects the minimum guaranteed interest only.
3. The green line is a VA-GMWB with a 8% annual simple interest rate for maximum 20 years, annual market value step-up is stacked²⁰

Figure 14: Asset Paths for 90% certainty of outcome for selected accumulation vehicles



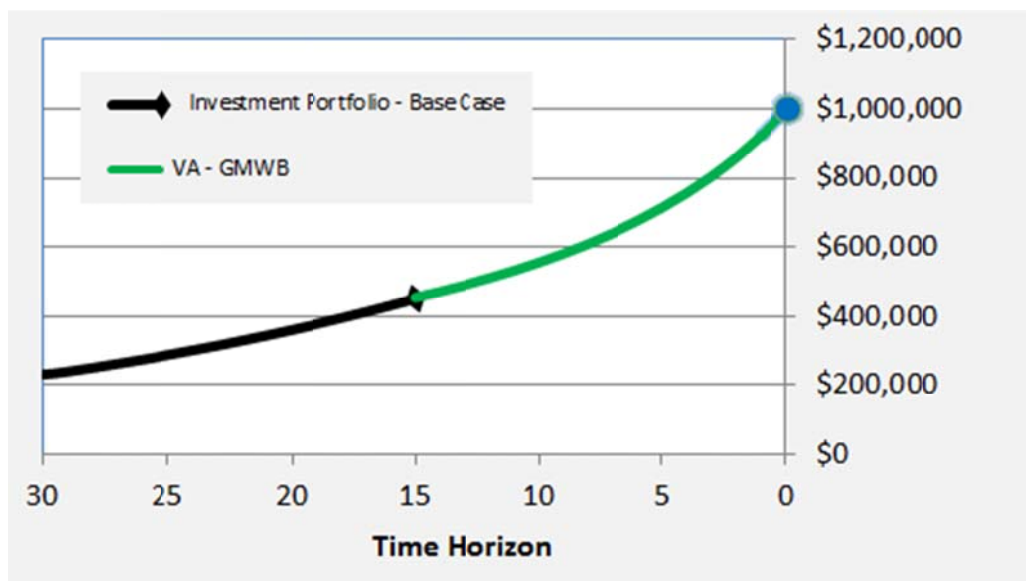
¹⁹ A higher rounded top two years prior to reaching the \$1 million target, is not an error: If you are very close to target (three years or less) and holding this investment portfolio, you need a larger dollar amount than the final target amount to cover the unlucky scenario. Alternately, you can shift all assets to cash once you reach the target amount two or three years prior to the target time horizon.

²⁰ In this case, we used the VA-GMWB offered by Allianz Insurance, called “Vision Income Protector”

We observe that the green line is the lowest of the three accumulation vehicles. Therefore, this particular VA-GMWB has the highest capital accumulation efficiency because you would need the smallest initial capital to reach the target with 90% certainty.

Multiple accumulation vehicles can be combined serially, as depicted in Figure 15. This chart displays the initial assets required to accumulate \$1 million at the end of 30 years using a combination of an investment portfolio (base case) for the first 15 years and then a VA-GMWB for the final 15 years. This type of depiction can help to further optimize different accumulation vehicles by picking and choosing each one's best parts for a particular time slice. The compound annual growth rate of this combination over this 30-year time horizon is about 5.0%. This is significantly larger than that of the investment portfolio at 3.9%.

Figure 15: Assets paths for serial accumulation vehicles for 90% certainty of outcome



If your purpose is to accumulate a **non-specific dollar amount at a given future target date**, then the next section (the addendum) provides the optimum asset allocation based on the non-Gaussian aftcast of all years since 1900.

Addendum: Optimum Asset Mix for Sur-Target Assets:

This paper was written to analyze strategies and products to achieve a specific target dollar amount in the portfolio for retirement income. The optimum asset allocation described earlier, which is designed for 90% certainty, serves that purpose.

However, there are cases where an investor has already met his target dollar amount in his portfolio to provide him his desired retirement income. That can happen for many reasons such as, being lucky with investments, receiving an inheritance, unexpected winnings etc. Also, by the mere fact that, if your design is for 90% certainty, historically, in 90% of the time, you will reach your target date sooner.

Once your assets exceed your target dollar amount to finance your retirement needs, the first step is to review your retirement lifestyle expenses to make sure that the original target is still valid. After that is reconfirmed, then any assets beyond that target are “surtarget”. Surtarget assets, i.e. assets that are over and above what you need for retirement, do not need to be invested with the same strict certainty of outcome. They can be invested to reflect the optimum asset mix is for the **median** outcome.

Table 20 depicts the non-Gaussian optimum asset allocation between equities and fixed income. This is based on maximizing the median portfolio value for surtarget assets. These figures are only for accumulation portfolios with no withdrawals. If there are any withdrawals such as in a pension fund, a foundation, a trust fund, or a retirement portfolio, these optimums will be different.

Table 20: Optimum asset mix, optimized for median portfolio value for surtarget assets:

Time Horizon	Optimum Asset Mix		
	Equity Proxy DJIA	Equity Proxy S&P500	Equity Proxy SP/TSX
10 years	50 / 50	40 / 60	60 / 40
20 years	50 / 50	60 / 40	70 / 30
30 years	60 / 40	70 / 30	60 / 40
40 years	80 / 20	100 / 0	60 / 40

Keep in mind; these figures are based on the last 111 years of market history. Future outcomes will be different. Also, investor’s personal risk tolerance overrides any of the mathematically optimum figures cited above.

About Aftcast.com

Aftcast.com provides research in the area of retirement income products to its clients. The research is based on non-Gaussian philosophy using actual market history. It helps its clients to better understand the behavior and impact of retirement income products under various, non-simulated, historical market environments. It provides the intelligence to its clients to make more informed decisions to manage and market their existing and planned retirement income products.

This report was researched and authored by Jim Otar, CFP, CMT, BASc, MEng, who is the founder of aftcast.com. Also, valuable editorial feedback was graciously provided by Kerry Pechter of Retirement Income Journal.

For your comments and feedback, or to learn more about aftcasting, please visit www.aftcast.com or send an email to jim@retirementoptimizer.com