Determinants of Growth in Distribution Portfolios: A Non-Gaussian Analysis

A White Paper from Aftcast.com

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Disclosure: Throughout this paper, terms "successful", "unsuccessful", "failure", "certainty", "likelyhood", "likely", "unlikely" and any such similar words refer only to statistical outcomes of the market history since 1900. Future outcomes will likely be different.

There is no assurance or certainty that any type of investment or investment strategy can guarantee a specific outcome or results.

First Draft, December 5, 2011
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Executive Summary:

We originally wrote extensively about the determinants of portfolio growth in "Unveiling the Retirement Myth – Advanced Retirement Planning based on Market History", a 525-page textbook[i] published in 2009. There, it was spread over eight chapters. This whitepaper combines and updates that into a more concise format.

All calculations are updated to reflect the market history until the end of 2010. The background information, our calculation methodology and our findings are described in the following pages.

We did not include any tax consequences in our analysis. We only looked at portfolio performances, excluding tax implications, if any.

Summary of Findings:

In our analysis, we use the entire 111-years of market history starting in 1900 and ending at the end of 2010. We include the effects of asset allocation, sequence/volatility of returns, inflation, as well as other factors such as withdrawal rates, rebalancing frequency, portfolio costs and the potential alpha that might be produced by better management of investments.

The methodology and basis of these conclusions are described in the following pages. In our aftcasting, the equity proxy for the entire analysis is the S&P500 index return, unless indicated otherwise. Fixed income net returns (after all expenses) are historical 6-month CD rate plus 0.5%, which corresponds to approximately a fixed income portfolio of five to seven year maturity.

The most important determinant is the combination of sequence and volatility of returns. The next important factor is asset allocation at lower withdrawal rates. As withdrawal rates increase and portfolio life shortens, the importance of asset allocation diminishes and is replaced by the effect of inflation.

Table 1 and Figure 1 summarize our findings.

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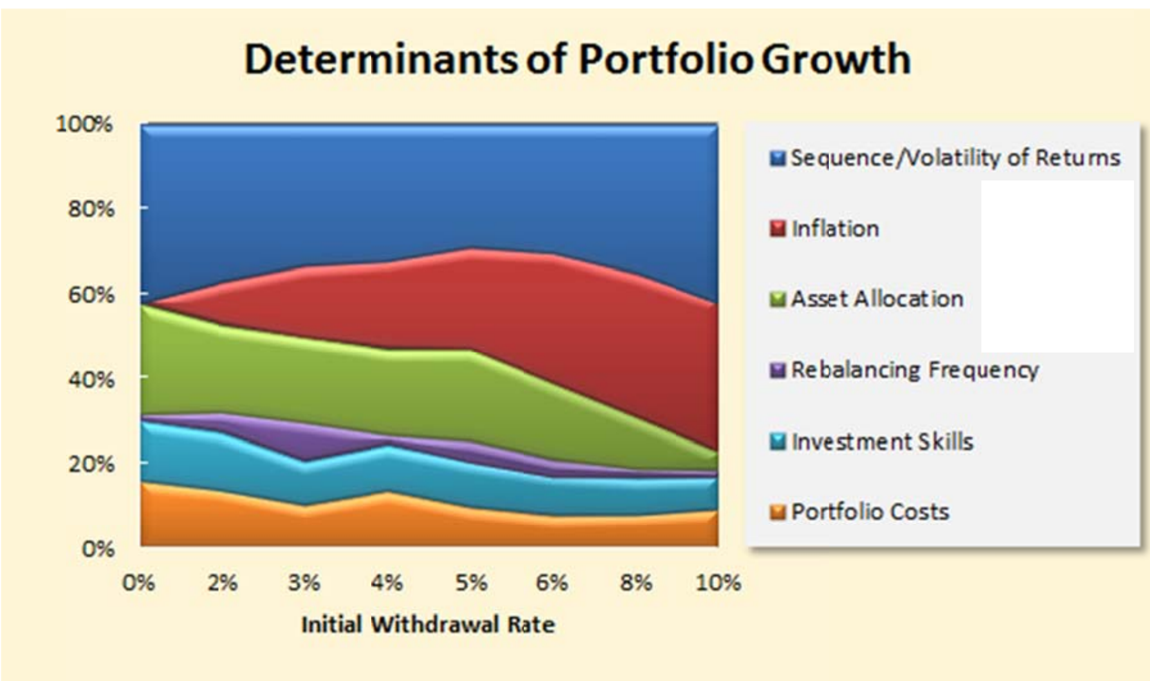
Table 1: Determinants of a Distribution Portfolio's Growth

	Initial Withdrawal Rate							
	0%	**2%**	**3%**	**4%**	**5%**	**6%**	**8%**	**10%**
	Determinants of Portfolio Growth							
Luck Factor:								
Sequence/Volatility of Returns	42%	37%	33%	32%	29%	30%	35%	41%
Inflation	0%	10%	17%	21%	24%	31%	34%	36%
Manageable Factors:								
Asset Allocation	26%	21%	20%	20%	21%	17%	11%	4%
Rebalancing	2%	5%	9%	2%	5%	4%	3%	1%
Fund Management Skills	14%	13%	11%	11%	11%	10%	8%	9%
Portfolio Costs	16%	14%	10%	14%	10%	8%	9%	9%

Note: All figures are rounded.

Figure 1: Determinants of a Distribution Portfolio's Growth



The “Brinson” Study

The first serious look at this topic was the study by Gary P. Brinson, Randolph L. Hood, and Gilbert L Beebower in 1984. In this research, the authors analyzed data from ninety-one large corporate pension plans with assets of at least $100 million over a 10–year period beginning in 1974. In the literature, it is generally referred to as the “Brinson Study”. Subsequently, the analysis was expanded to include an additional ten years of data and "Determinants of Portfolio Performance II" was published in the Financial Analysts Journal, January/February 1995.

Their conclusion was that the components of the *difference in success* of a portfolio are: Asset allocation: 93.6%; Security selection 2.5%; Other: 2.2%; Market timing 1.7%.

Since then, many in the financial planning profession try hard to make investors believe that asset allocation is the Holy Grail of investing. When a new account is opened, the first thing a client does is to fill out a risk–assessment questionnaire. Based on the client’s answers, he or she is then pigeonholed into one of four or five investment portfolios. We do not believe that this is the right approach.

Here is where we see a problem: The findings of the Brinson study cannot be transferred, scaled or applied to individual retirement portfolios for the following reasons:

1. The dynamics of cash flow in a pension fund are entirely different from the dynamics of cash flow in an individual retirement account. A pension fund has a continuous inflow of money over time. In an individual retirement account, inflow of money occurs mainly during working years. After retirement, there is usually no more inflow but only outflow. Mathematically, a pension fund is an “open–perpetual” system; an individual retirement account is a “closed–finite” system.
2. When and if there is a shortfall in a pension fund, then contributions are increased to meet this shortfall. On the other hand, with an individual retirement savings, there is usually no such opportunity.
3. In an individual retirement account, once the withdrawals start, the adverse effect of “reverse dollar–cost–averaging” becomes important. In a pension fund, since there is a continuous inflow of money concurrently, this effect is insignificant.
4. In an individual account, inflation is important. Withdrawals must be increased over time to maintain the same purchasing power for the retiree. In pension funds, there is no such concern; as inflation goes up, salaries and

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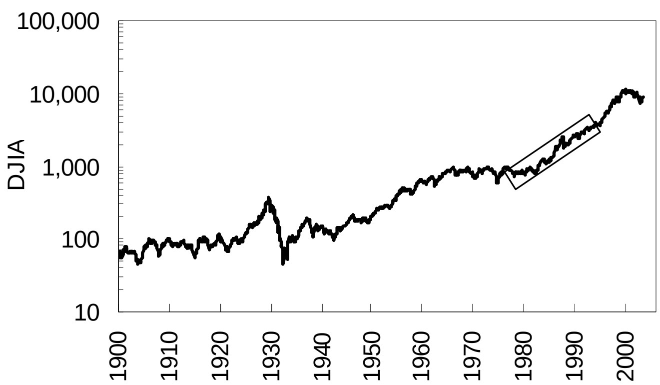
pension contributions increase. Also, many pension funds have limits and constraints on how the retirement payments are indexed. Individual retirees holding their own saving accounts do not have that choice; their expenses must be met.

5. The time span of the Brinson study is twenty–years. In the historical context, it is not only too short, but it covers a single secular bullish trend, arguably the "luckiest" 20-year time period over the entire twentieth century. Such a short time frame will miss significant adverse events that are present exclusively in secular bear or secular sideways market trends.

Figure 2: Time period covered in the Brinson study



Our Approach

The Brinson study is a valuable research work. There is no doubt that asset allocation is important for a pension fund's success, subject to the limitations mentioned above. However, we believe it is abused by many in the financial industry by extrapolating its findings to individual investment/retirement accounts.

Ideally, the proper asset allocation reduces the *volatility of returns* to an acceptable level, such that its owner stays invested through thick and thin. We have no problem with that. However, the *sequence of returns* and *inflation* have a far greater impact on the outcome and it is not covered in the Brinson study.

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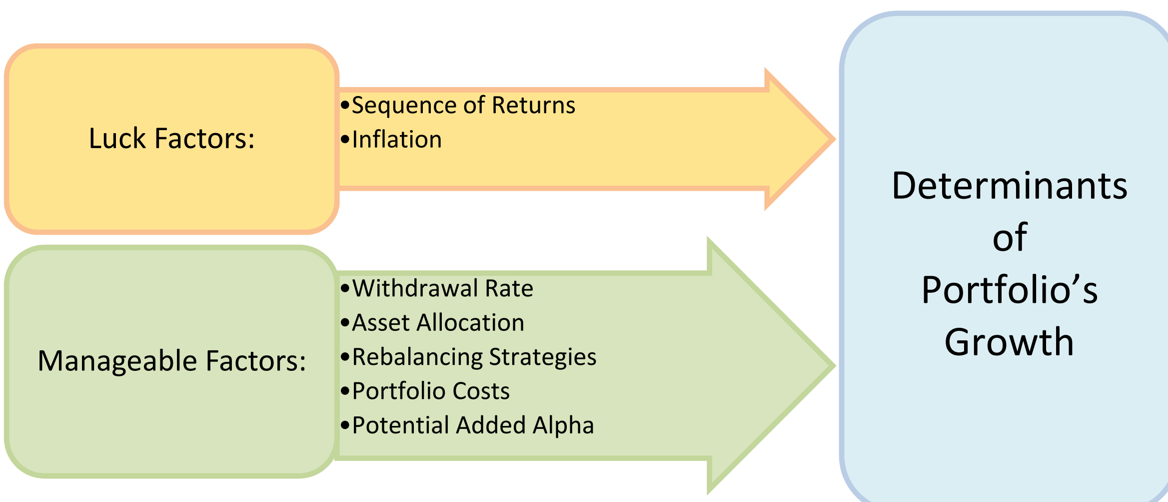
In our analysis, we use the entire 111-years of market history starting in 1900 and ending at the end of 2010. We include the effects of asset allocation during that entire time period, not just 10 or 20 years. In addition, we also include other factors not included in the Brinson study, such as, withdrawal rates, the effect of sequence of returns, inflation, rebalancing frequency, portfolio costs and the potential alpha produced by better management of investments.

Our analysis does not use any forecasts based on Gaussian methods. We do not use man-made simulators of any kind, generally known as Monte Carlo simulators. We use actual market history which we call "aftcasting". If you are not familiar with the definition of "Gaussian", please see Appendix A for further description and our criticism of it.

We call the type of factors that are outside of the control of the investor, the "luck factor". It has two components: the sequence of returns and inflation.

Generally, all other factors can be either managed by the investor or by the investment (or portfolio) manager. We call these "manageable factors"

Figure 3: Determinants of a Portfolio's Growth



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Our Methodology

The methodology for calculating the contribution of each component is explained in detail in the following pages. However, before going into each component, let's review the concept of aftcasting, the engine that provides us with the historical outcomes.

Aftcasting displays the outcome of all historical asset values of all portfolios since 1900 on the same chart, as if a scenario starts in each one of the years between 1900 and 2000. It gives a bird's-eye view of all outcomes. It reflects the sequence of returns exactly as it happened in history. It is developed by the author of this whitepaper when writing his original book[ii] on retirement planning in 2001. It provides the success and failure statistics with exact historical accuracy, as opposed to man-made simulation models because it includes the actual historical equity performance[iii], inflation rate[iv] and interest rate[v], as well as the actual historical sequencing of all these data sets.

Let's look at an example: Bob, 65, is just retiring. He plans on withdrawing $15,000, indexed annually to inflation, until age 95. His primary concern is sustainability of his income stream for life. In his investment portfolio he has $350,000 with an asset mix of 40% equities and 60% fixed income.

The aftcast of this scenario is depicted in Figure 4. On this chart, we see the thin, black aftcast lines. There is one line starting at the left vertical axis for each year since 1900. There are 40 years of data on each aftcast line for all starting years before 1972. After 1971, each aftcast line ends at the end of year 2010. Thus, there are 3675 data points that reflect the exact, actual market history which is exactly in-line with realistic correlations and patterns of performance of equities, bond yields, interest rates and inflation.

We define the bottom decile (bottom 10%) of all outcomes as the "unlucky" outcome, the top decile (top 10%) as the "lucky" outcome. The blue line indicates the median outcome where half of the scenarios are better and half are worse. In this example, the probability of depletion by age 95 happens to be 34%.

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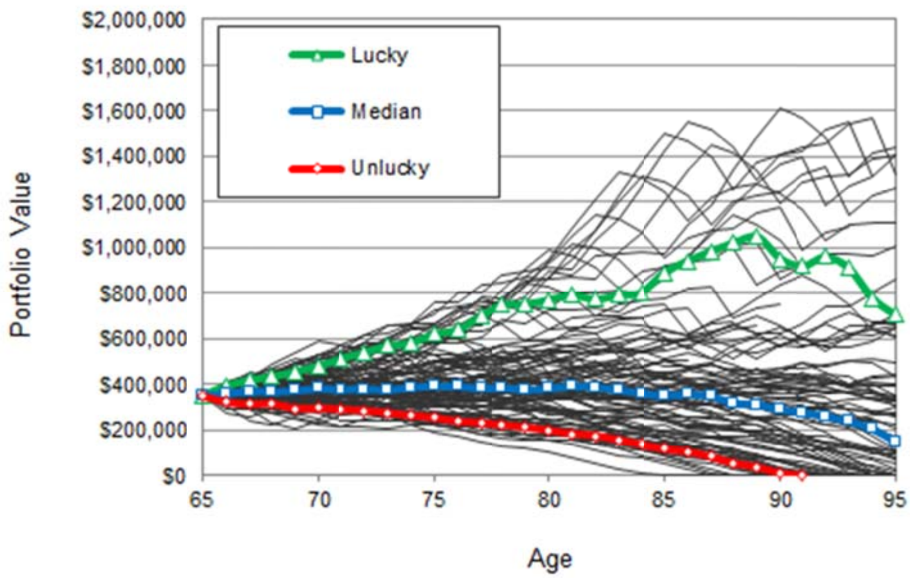
Figure 4: The aftcast of fixed $15,000 annual withdrawals, indexed to inflation, from an investment portfolio, starting capital of $350,000



Aftcasting only shows what would have happened in history given a specific set of input data. We do not use it to predict anything. We agree with the idea that past events will be repeated in the future. We are not interested in what happened in a specific year in history, other than to demonstrate it in the examples.

However, we are very interested in *the frequency, the size and the persistency* of extreme events (that is, market and/or inflation events) that happened in the past in real life, i.e. a non-Gaussian framework. These extremes are our starting point for designing a robust retirement plan for our clients.

With this background, we can now start analyzing each component that affects growth in distribution portfolios.

The Sequence/Volatility of Returns

In a distribution portfolio, the sequence of returns is the most important component of the luck factor. It is the direction and persistency of the volatility of returns. Mathematically, it can be defined as the first time-derivative of the volatility of returns.

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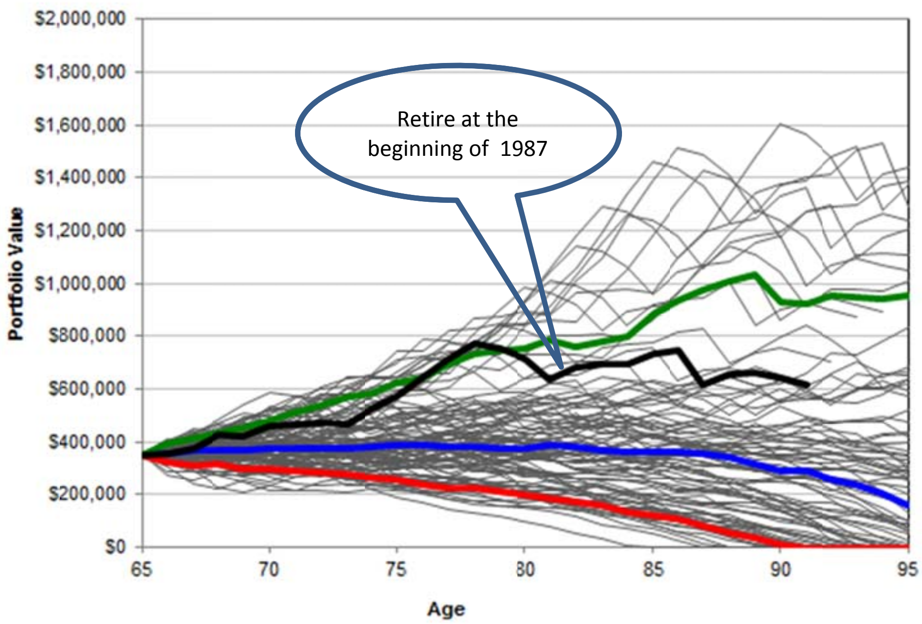
Volatility of returns by itself does not damage a distribution portfolio. It is the sequence of returns that can do the irreparable damage. For example, if you were to retire in 1987, you would have experienced a significant volatility of returns; a 30% drop in the equity index during October and November of that year. However, after this large drop, equity markets recovered quickly and this adverse volatility was not accompanied by an adverse sequence of returns. Consequently, there was no damage to the portfolio's longevity. Figure 5 shows the retirement year 1987 in heavy black line.

Figure 5: The effect of adverse volatility of returns, if not accompanied by an adverse sequence of returns. Retirement starting year 1987 is indicated in heavy black



Let's look at how sequence of returns works over a four year time horizon. We have two scenarios: In the first scenario, the investor is lucky in the beginning; his portfolio grows by 20% in each of the first two years and then it declines by 10% in each of the final two years. This is the "good start" portfolio.

In the second scenario he is unlucky in the beginning; his portfolio declines by 10% in each of the first two years and then grows by 20% in each of the final two years. This is the "bad start" portfolio.

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Table 2: Sequence of Returns

Year	Annual Growth	
	Good Start	**Bad Start**
1	20%	-10%
2	20%	-10%
3	-10%	20%
4	-10%	20%
Average Growth:	5%	5%

In each case, the average annual growth is identical, it is 5%. Let's see how these scenarios affect the total growth. First, we look at an accumulation portfolio. The investor starts with $100,000 initial capital and no money is added to or withdrawn from the portfolio. The table below (Table 3) shows the portfolio value at the end of each year:

Table 3: Sequence of Returns, accumulation

Year	Portfolio Value- Accumulation (initially $100,000)	
	Good Start	**Bad Start**
1	$120,000	$90,000
2	$144,000	$81,000
3	$129,600	$97,200
4	$116,640	$116,640
Total Growth:	16.64%	16.64%

In both scenarios, good or bad start, the total growth during the four-year time period is identical, 16.64%. **If no money is distributed, then the sequence of returns has no effect on the final outcome.**

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Now, let's look at a distribution portfolio. The same investor starts with the same portfolio, experiences the same good-start/bad-start scenarios, but in this case he withdraws $5,000 at the end of each year. Here is the portfolio value at the end of each year:

Table 4: Sequence of Returns, distribution

Year	Portfolio Value - Distribution (initially $100,000, $5,000 withdrawn at the end of each year)	
	Good Start	**Bad Start**
1	$115,000	$85,000
2	$133,000	$71,500
3	$114,700	$80,800
4	$98,230	$91,960
Total Decline:	1.77%	8.04%

When the investor had a good start, his portfolio's total decline -including his withdrawals- was 1.77%. When he had a bad start, his total decline over the four-year time period was much larger, 8.04%. This is the effect of the sequence of returns in a distribution portfolio.

Now, let's focus on the difference between the impact of the volatility of returns and the sequence of returns. In essence, the volatility of returns creates an adverse "reverse-dollar-cost-averaging" for the portfolio. History shows that this can reduce the portfolio life by up to 20% (in the worst-case). However, it can be easily fixed by creating a "cash" bucket, holding about five years of withdrawals in cash-like investments and short-term bonds. When withdrawals are made from this bucket only, then the effect of volatility of returns can be diminished significantly.

On the other hand, the effect of sequence of returns cannot be reduced with the bucket strategy because, once withdrawals exceed the sustainable amounts, then the time horizon of the *investor* and the time horizon of the *portfolio* disconnect. As a result, losses become permanent and irrecoverable, regardless of how long (or how short) the investor's time horizon is.

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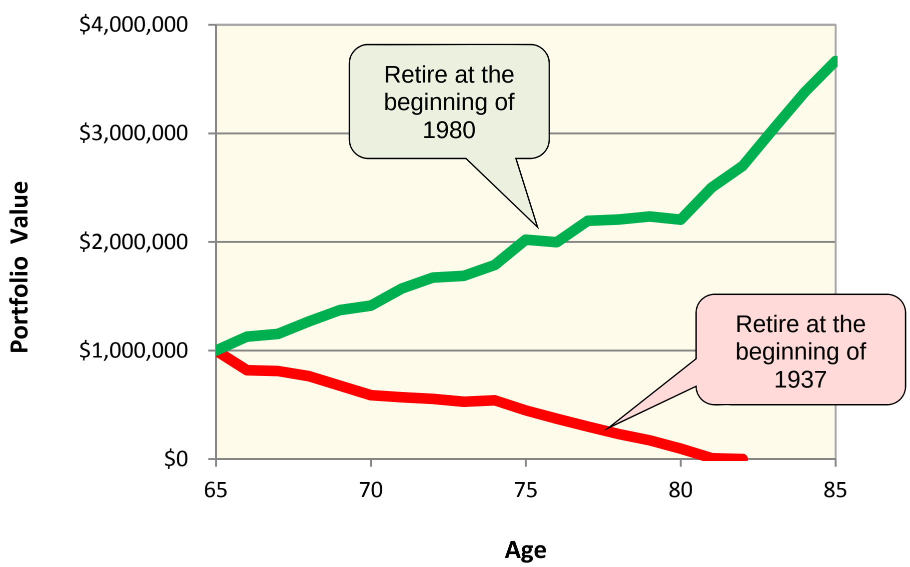
While it is not too difficult to analyze the effects of volatility of returns and the sequence of returns separately, it is beyond the scope of this article. For the purposes of calculating the luck factor, that distinction is unnecessary. Therefore, we combined the sequence and the volatility of returns as a single line item separated by a slash.

Figure 6 depicts the effect of starting retirement in 1937 (unlucky timing) versus 1980 (lucky timing). In both cases, the retiree starts with an initial capital of $1 million, 40/60 asset allocation, rebalanced annually, and the initial withdrawal amount is $50,000 at age 65, indexed to inflation each year.

The lucky investor retired in 1980. At age 85, his portfolio grew to nearly $4 million. The unlucky investor retired in 1937. His portfolio ran out of money at age 81. Since all other factors are identical, the luck factor is the only determinant of this huge difference in the outcome.

Figure 6: The importance of luck with respect to timing of the retirement



How do we calculate the effect of the sequence/volatility of returns in a distribution portfolio? Here are the steps:

1. Isolate and exclude the effect of the variability of inflation from secular trends. We do that by using a fixed "average" inflation rate during retirement. This leaves us with variations in the sequence of returns/volatility only.

2. Calculate the asset value of the portfolio over time for all years since 1900. We define the top 10% of all outcomes as the "lucky" outcome and the bottom 10% of all outcomes as the "unlucky" outcome.
3. Calculate the median outcome, where half of the outcomes are better and half are worse.
4. Calculate the compound annual return (CAR) of the lucky, unlucky and the median portfolios.
5. Finally, the effect of the sequence of returns/volatility is half of the difference between the CAR of the lucky and unlucky portfolios divided by the CAR of the median:

$$LF = \frac{(CAR_{90} - CAR_{10})}{2 \times CAR_{50}} \times 100\% \qquad \text{(Equation 1)}$$

where:

LF is the luck factor

CAR_{90} is the compound annual return of the lucky (top decile) portfolio

CAR_{10} is the compound annual return of the unlucky (bottom decile) portfolio

CAR_{50} is the compound annual return of the median portfolio

The luck factor measures the average difference of the compound annual returns between the extreme outcome and the median outcome, expressed in percentage.

Example: 1

Dan is 65 years old, just retiring. He has $1 million in his portfolio and needs $60,000 each year, indexed to inflation. His asset mix is 40% equity and 60% fixed income, rebalanced annually.

On the equity side, he expects the index return. On the fixed income side, he expects a return of 0.5% over and above the historical 6–month CD rates after all management fees. Using 3.2% annual increase of withdrawals to reflect the historical average inflation, we calculate his luck factor resulting from the sequence of returns.

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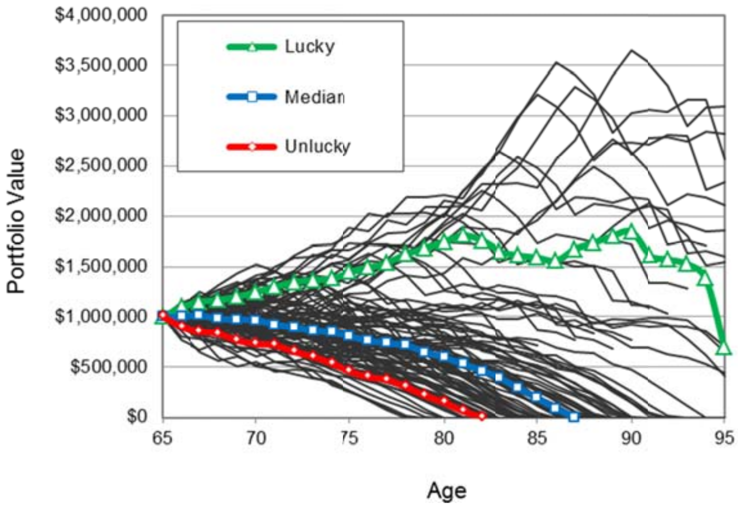


The lucky, unlucky and median portfolios are indicated on the chart. We calculate the compound annual returns for the lucky, unlucky and median portfolios as 8.00%, 2.83% and 5.46%, respectively. Plug in these numbers into Equation 1 and calculate the luck factor resulting from the sequence/volatility of returns:

$$\text{Luck Factor attributable to sequence/volatility of returns} = \frac{8.00\% - 2.83\%}{2 \times 5.46\%} = 47.3\%$$

Similarly, we calculate the luck factor resulting from the effect of sequence/volatility of returns for various withdrawal rates. Table 5 shows the results. In all cases, the asset mix is 40% equity and 60% fixed income.

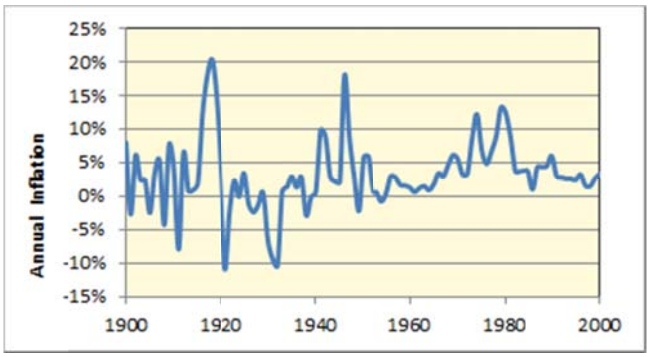
Table 5: The impact of the sequence/volatility of returns for various withdrawal rates

Initial Withdrawal Rate							
0%	**2%**	**3%**	**4%**	**5%**	**6%**	**8%**	**10%**
Effect on Portfolio Growth Rate							
46.7%	46.3%	46.5%	51.8%	50.2%	47.3%	62.2%	69.1%

Sequence of Inflation

Inflation, or more accurately, the sequence of inflation is the second most important component of the luck factor[vi]. Even a short-term increase of inflation compels the retiree to withdraw higher and higher amounts from his portfolio for the rest of his life just to maintain his purchasing power. This can deplete a retirement portfolio prematurely.

Figure 7: Inflation since 1900 (Source: U.S. Department of Labor, BLS)



The target inflation rate set by the Federal Reserve is currently at 2%. A large portion of retirement plans are designed for 2% to 3% annual indexation of income. On the other hand, the history shows that the inflation rate was less than 3% only in 52% of the time. Therefore any retirement plan that "assumes" an average of 3% (or lower) inflation is incongruent with the historical experience. There is an (almost) even chance that inflation might be a higher than that, causing the portfolio to deplete sooner than planned.

This is where the **sequence of inflation** comes into play: You do not need chronic high inflation lasting a long time to suffer the adverse effects of the sequence of inflation. If a retirement plan is designed for a "normal" inflation rate, only a few years of "high" inflation can reduce the portfolio life significantly.

The following chart (Figure 8) illustrates a hypothetical case: This particular retirement plan is designed to start at age 65 with an initial capital of $1 million. The plan assumes an 8% average growth rate of the portfolio. The retiree needs $66,000 / year, indexed to 3% annually until age 95. With these assumptions –albeit aggressive- the "forecast" indicates an uninterrupted income stream until age 95. This is designated on the graph as "Normal CPI Sequence".

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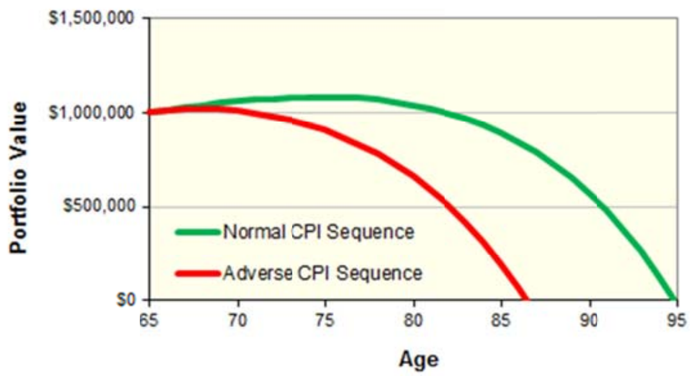
Now consider that this retiree experiences a higher than "normal" CPI for the first three years of retirement. Between ages 65 through 67 (inclusive), the annual CPI jumps to 10% and then reverts back to 3% at age 68 for the rest of his life. This is designated as "Adverse CPI Sequence" on the chart. We observe that the portfolio longevity is reduced from age 95 to age 86, i.e. a 30% reduction over the original 30-year time horizon.

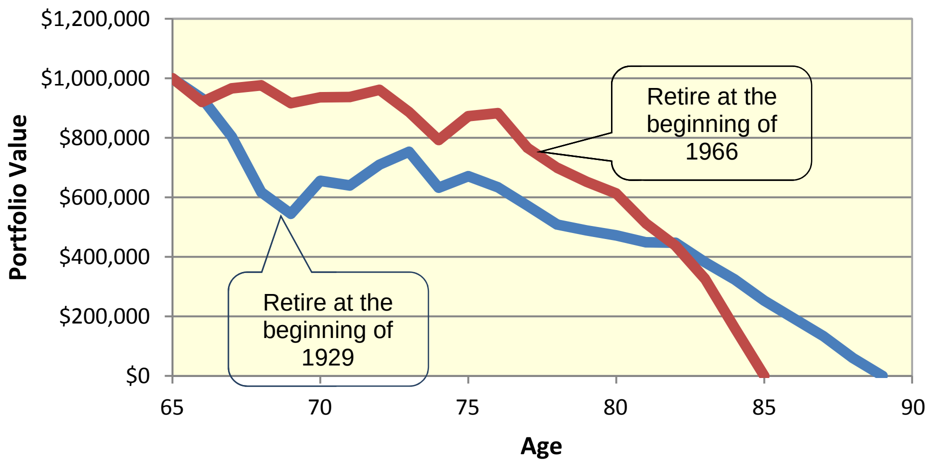
Figure 8: Sequence of Inflation, hypothetical case



Let's look at two examples from the last century. Consider a retiree with an asset mix of 40/60 equity/fixed income and a 6% initial withdrawal rate. Here, we use historical dividend rates and assume no management fees. If this person were to retire at the beginning of the market crash of 1929, his portfolio would have lasted until age 89. On the other hand, if he were to retire in 1966, the beginning of a secular sideways market, his portfolio would have depleted at age 85, as shown in Figure 9.

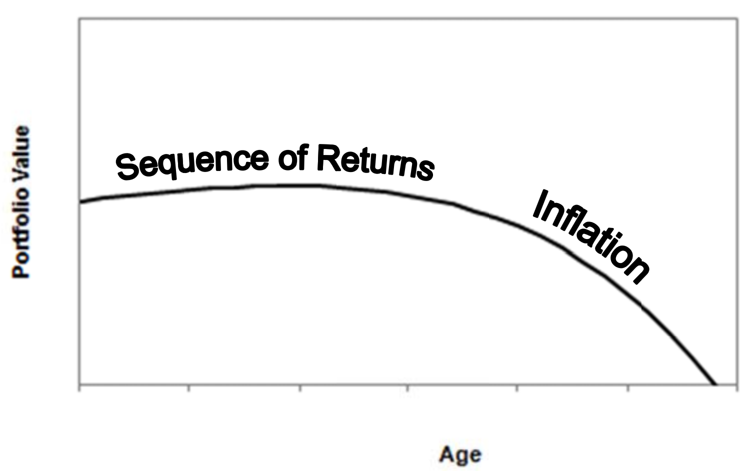
The high inflation between 1966 and 1981 would have forced the retiree to withdraw more and more income, eventually depleting his portfolio. The net effect of this was worse than the secular bearish trend that started in 1929, when equities lost about 80% of their value.

Figure 9: High inflation can shorten portfolio life more than the worst market crash.



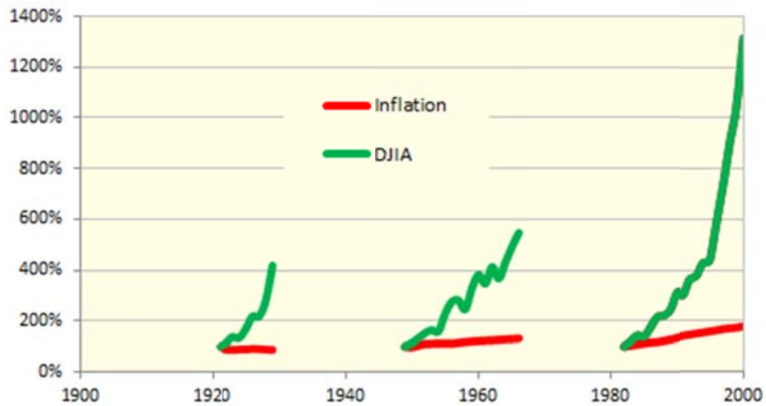
Generally, the sequence of returns impacts the portfolio in the early years and the inflation impacts it in later years. If the portfolio value drops sharply in the early years, it is generally because of the adverse sequence of returns. If the drop is in later years, it is almost always because of adverse sequence of inflation which might have been triggered several years prior. Figure 10 shows the areas of the standard retirement plan where sequence of returns plays an important role and where sequence of inflation plays an important role on portfolio longevity.

Figure 10: The influence of the Sequence of Returns and Inflation on distribution portfolio



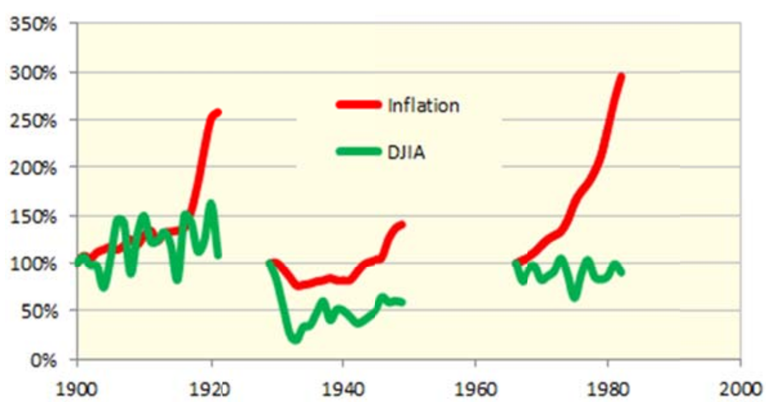
More often than not, the sequence of returns and sequence of inflation are cumulative. During the secular bullish trends (1921-1928, 1949-1965, 1982-1999), which occupy 43% of the last century, the growing equity index easily covered any adverse effects of inflation. Figure 11 depicts this observation:

Figure 11: The performance of DJIA (index only) and inflation during secular bullish trends



However, during the remaining 57% of the time, the markets were either in secular bearish or secular sideways trends. The equity index was never able to overcome the effects of inflation for the entirety of these time periods. Figure 12 depicts this observation:

Figure 12: The performance of DJIA (index only) and inflation during all secular bearish/sideways trends



How can we calculate the luck factor created by the effect of inflation in a retirement portfolio? We do it similar to calculating the effect of the sequence/volatility of returns. Here are the steps:

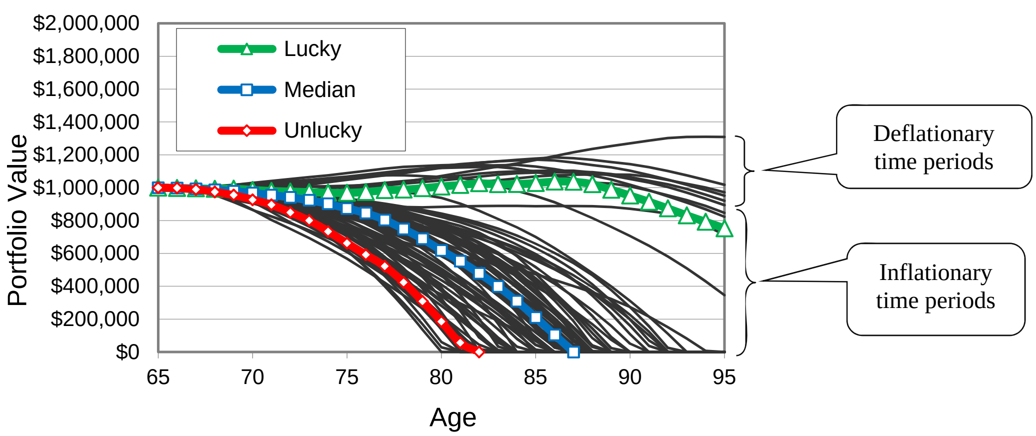
1. Isolate and exclude the effect of the sequence/volatility of returns. We do that by using a fixed "average" portfolio growth rate in the aftcast. This leaves us only with the historical inflation rates that vary from year to year.
2. Calculate the compound annual return (CAR) of the lucky, unlucky and the median portfolios.
3. Calculate the luck factor due to inflation using Equation 1.

Example: 2

Marco is 65 years old, just retiring. He has $1 million in his portfolio and needs $60,000 each year, indexed to inflation. His asset allocation is 40% equities and 60% fixed income, rebalanced annually.

Marco assumes that he will receive the index return, 6.7% (between the years 1900 and 2010) after all management fees. As for the fixed income side, the average return was 5.1% for the same time period.

Therefore, we calculate the average return for a 40/60 asset mix portfolio as 5.74% (40% of 6.7% equity growth and 60% of 5.1% fixed income growth). We use that as the portfolio growth rate and we index withdrawals to actual CPI annually.



The lucky, unlucky and median portfolios are indicated on the aftcast chart above. We calculate the compound annual returns for the lucky, unlucky and median portfolios as 8.04%, 2.77% and 5.50%, respectively. Plug these numbers into Equation 1 and calculate the luck factor that is attributable to inflation:

$$\text{Luck Factor attributable to inflation} = \frac{8.04\% - 2.77\%}{2 \times 5.50\%} = 47.9\%$$

Similarly, we calculate the luck factor resulting from the effect of inflation for various withdrawal rates. Table 6 shows the results. In all cases, the asset mix is 40% equity and 60% fixed income.

Table 6: The impact of inflation for various withdrawal rates

Initial Withdrawal Rate							
0%	**2%**	**3%**	**4%**	**5%**	**6%**	**8%**	**10%**
Effect on Portfolio Growth Rate							
0.0%	12.3%	23.3%	33.2%	40.9%	47.9%	59.2%	59.5%

Asset Allocation

When people cite the Brinson study, they might say something like "asset allocation is the single largest contributor to a portfolio's success. It is much more important than security selection. In fact, one study concluded that asset allocation accounted for over 90% of the difference in a portfolio's investment return."

Here is the reality: Take a 65-year old investor, retiring this year. He wants to plan until age 95. His retirement savings are valued at one million dollars. He needs to withdraw $60,000 each year, indexed to actual inflation. On the equity side, he expects an average of 2% dividend yield, pays 0.5% management fees.

Let's look at how his portfolios would have performed if he were to start his retirement in any of the years between 1900 and 2000. We aftcast six different asset mixes:

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Table 7: The impact of asset allocation for various withdrawal rates

Asset Mix (Equity / Fixed Income)	Probability of Depletion by Age 95	Median Portfolio depleted at Age
100% Equity	68%	87
80 / 20	67%	87
60 / 40	74%	87
40 / 60	78%	86
20 / 80	91%	86
100% Fixed Income	95%	87

We see on Table 7 that, neither the median portfolio life, nor the probability of depletion, improved significantly by changing the asset allocation.

While asset allocation is an effective tool to limit the volatility of returns, its effect on the sequence of returns is generally insignificant in decumulation portfolios.

A decumulation portfolio is defined as a portfolio where the historical median portfolio value (in an aftcast) depletes before the age of death.

With that observation, we can now measure the impact of asset allocation. We figure out the difference in compound annual returns (CAR) of the median portfolio for the asset mix with the best and the worst CAR as described in Example 3.

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Example 3

Bob, 65, is just retiring. He has $1,000,000 savings for retirement; he needs $30,000 each year, indexed to inflation. His equities grow the same as the S&P500 index. He rebalances his asset mix annually if equities deviate by more than 3%.

Based on the market history, the compound annual return (CAR) of the median portfolio for various asset mixes are as follows:

	Asset Mix (Equity / Fixed Income)								
	0/100	20/80	30/70	40/60	50/50	60/40	70/30	80/20	100/0
CAR, (median)	4.98%	4.92%	4.98%	5.07%	5.28%	5.41%	5.16%	5.10%	4.22%

For this example, based on market history, the highest growth rate was at 5.41% and the lowest was 4.22%. If Bob makes the worst asset allocation decision, the maximum penalty is a 1.19% difference in CAR in absolute terms. In relative terms, the difference is 28.2%, calculated as 1.19% divided by 4.22%.

Table 8 indicates the impact of asset allocation for various withdrawal rates.

Table 8: The impact of asset allocation on portfolio growth

Initial Withdrawal Rate							
0%	**2%**	**3%**	**4%**	**5%**	**6%**	**8%**	**10%**
Effect on Portfolio Growth Rate							
29.0%	25.9%	28.2%	33.0%	36.8%	27.5%	19.3%	7.2%

Rebalancing Frequency

When it comes to rebalancing, many investment professionals believe often is better. Rebalancing is done –supposedly- to reduce the portfolio volatility. Does frequent rebalancing really decrease volatility? How does this impact portfolio longevity? Let's observe the history.

Volatility has two components. The first component is the short–term random fluctuations. Every second, every minute, every day, some event happens somewhere in the world that influences investor psychology. As investors make trading decisions, markets move up or down. If we agree with the notion that price movements within a one year time horizon are mostly random, then we cannot expect a significant reduction in volatility by rebalancing more frequently than annually. So, you can rebalance as often as you want, even daily, and that won't reduce the random volatility.

The second component of volatility occurs over the longer term. Markets respond to the collective expectation of investors and a trend forms. Rebalancing can reduce volatility only if it is done after an *observable trend*. Our analysis shows that the 4–year U.S. Presidential election cycle is the shortest market cycle with an observable trend that we can work with. Example 4 shows the impact of rebalancing frequency.

Example 4:

Steve, 65, is retiring this year. He has put aside $1 million for his retirement, 40% equity and 60% fixed income. He needs $50,000 income each year, indexed to inflation. He takes his withdrawal from the fixed income portion of his portfolio.

Let's see the impact of rebalancing for different market trends.

<u>Retiring into a Bearish trend – 1929:</u>

The chart below shows Steve's portfolio value if he had retired at the beginning of 1929, the beginning of a secular bear market. At the market bottom of 1932, Steve's portfolio experienced a smaller loss when rebalanced every four years than if he were to rebalance every year. The portfolio that was rebalanced every four years provided Steve with 28 years of income. On the other hand, if rebalanced annually, the portfolio would run out of money after 21 years. Rebalancing every four years on the Presidential election year increased the portfolio life by a respectable 38%.

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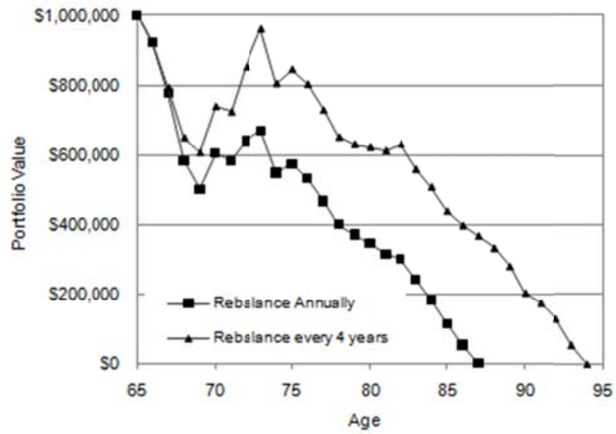
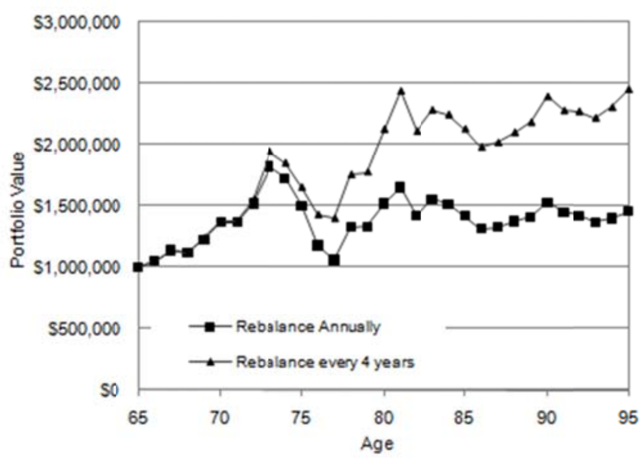


Retiring into a Bullish Trend – 1921:

The chart below shows the portfolio value if Steve had retired in 1921, the beginning of the first secular bull market of the last century. At the end of 30 years, Steve was one million dollars richer if he rebalanced every four years at the end of the U.S. Presidential election year than if he were to rebalance annually. The volatility was about the same for either.



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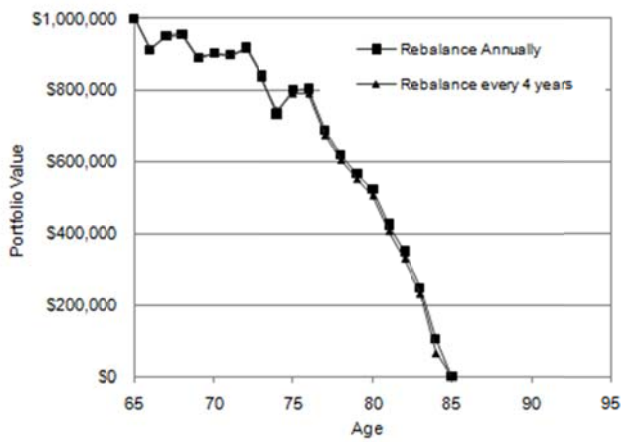
Retiring into a Sideways Trend – 1966:

The chart below shows Steve's portfolio value if he had retired at the beginning of a secular sideways trend that prevailed between 1966 and 1981. It demonstrates that there was no perceivable difference in the portfolio value when rebalanced every four years on the Presidential election year as opposed to rebalancing annually. The portfolio volatility was essentially identical.



The aftcast shows:

- The volatility was about the same whether the portfolio was rebalanced annually or once every four years on the Presidential election year.
- In secular bull markets, rebalancing too often stunted the portfolio growth.
- In secular bear markets, rebalancing too often compounded losses.
- In sideways markets, it did not matter how often you rebalanced. The portfolio life varied slightly at random.

Our analysis shows that rebalancing at the end of each Presidential election year gave the best results because doing so synchronized with the high point of this cycle. Rebalancing at any other frequency or at any other time within the cycle did not add as much value.

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We measure the impact of the frequency of rebalancing by observing the difference of the compound annual returns (CAR) of the median portfolios for both annual and Presidential cycle rebalancing scenarios.

For example, at 4% withdrawal rate the CAR of the median portfolio with annual rebalancing is 5.21%. When rebalancing is done once every four years at the end of the Presidential election year, then CAR becomes 5.41%. The impact of using the less frequent rebalancing is 3.8%, calculated as 5.41% less 5.21% divided by 5.21%.

Table 9 indicates the impact of rebalancing frequency on the CAR for a portfolio consisting of 40% equity and 60% fixed income. The rebalancing threshold is 3%, i.e. rebalancing occurs only if the asset mix deviates by more than 3%.

Table 9: The impact of rebalancing frequency on portfolio growth at various withdrawal rates

Initial Withdrawal Rate							
0%	**2%**	**3%**	**4%**	**5%**	**6%**	**8%**	**10%**
Effect on Portfolio Growth Rate							
1.9%	6.1%	12.2%	3.8%	8.5%	5.9%	5.8%	2.2%

Keep in mind; markets were in secular sideways trends during 50% of the last century. Therefore, when we average it over the century, it might seem like a small impact. However, it has a much larger positive impact in market extremes and we think this is valuable for the retiree.

Fund Management Skills - Added Alpha

While it is rare, exceptional fund managers can outperform the market for a period of time. This excess return over and above the index return is called alpha. In our analysis, to account for this potential outperformance, we use an alpha of 2%. We are not suggesting that a skillful manager can outperform the index by 2% year after year; but we use that as a possible upper limit for our calculation purposes.

First, we calculate the compound annual return (CAR) of the median portfolio with alpha equal to zero. Then, we do the same with alpha equal to 2%. In all cases, the asset mix is

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40% equity and 60% fixed income. We then express the impact of added alpha by observing the difference of the compound annual returns as a percentage.

For example, at 4% withdrawal rate the CAR of the median portfolio with index return is 5.21%. When alpha is increased to 2%, then CAR becomes 6.13%. The impact of added alpha is 17.7%, calculated as 6.13% less 5.21% divided by 5.21%.

Table 10 indicates the impact of fund potential management skills on the CAR.

Table 10: The impact of outperforming the index by 2%

Initial Withdrawal Rate							
0%	**2%**	**3%**	**4%**	**5%**	**6%**	**8%**	**10%**
Effect on Portfolio Growth Rate							
15.7%	16.8%	14.8%	17.7%	18.1%	15.1%	14.8%	14.0%

Portfolio Costs

Portfolio expenses can significantly reduce portfolio longevity or impede its growth over the long term. Like inflation, its effect is not readily apparent in a short period of time, but it adds up over the long term.

For the purposes of our analysis, we will roll all portfolio costs into a single fixed rate percentage. We use 1.5% of the value of equity holdings as portfolio costs. This includes everything; management fees, trading costs, portfolio expenses, commissions, account fees, and so on. If your portfolio costs are higher than this, its impact will be higher than what we calculated and depicted in our analysis here. Conversely, if they are lower, its impact will be lower than what we have in this analysis.

We first calculate the compound annual return (CAR) of the median portfolios first with alpha equal to zero. Then, we do the same with alpha equal minus 1.5% to reflect the portfolio costs on the performance. The impact of portfolio costs is the difference of the compound annual returns, expressed as a percentage.

For example, at 4% withdrawal rate the CAR of the median portfolio with index return is 5.21%. When portfolio costs are deducted, then aftcast shows the CAR of the portfolio as 4.07%. The impact of portfolio costs is 21.9%, calculated as 5.21% less 4.07% divided by 5.21%.

Table 11 indicates the impact of portfolio costs on the CAR for a portfolio consisting of 40% S&P500 and 60% fixed income.

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Table 11: The impact of paying 1.5% portfolio costs for the equity holdings

Initial Withdrawal Rate							
0%	**2%**	**3%**	**4%**	**5%**	**6%**	**8%**	**10%**
Effect on Portfolio Growth Rate							
17.7%	17.3%	14.4%	21.9%	17.3%	12.7%	15.0%	15.6%

Combining the Determinants of a Portfolio's Growth

Now, we can combine all these factors to calculate the determinants of a portfolio's growth. First let's summarize all factors that we have calculated so far in single table:

Table 12: The summary of impact on a portfolio's growth for various withdrawal rates

	Initial Withdrawal Rate							
	0%	**2%**	**3%**	**4%**	**5%**	**6%**	**8%**	**10%**
	Effect on Portfolio Growth							
Luck Factor:								
Sequence/Volatility of Returns	46.7%	46.3%	46.5%	51.8%	50.2%	47.3%	62.2%	69.1%
Inflation	0.0%	12.3%	23.3%	33.2%	40.9%	47.9%	59.2%	59.5%
Manageable Factors:								
Asset Allocation	29.0%	25.9%	28.2%	33.0%	36.8%	27.5%	19.3%	7.2%
Rebalancing	1.9%	6.1%	12.2%	3.8%	8.5%	5.9%	5.8%	2.2%
Fund Management Skills	15.7%	16.8%	14.8%	17.7%	18.1%	15.1%	14.8%	14.0%
Portfolio Costs	17.7%	17.3%	14.4%	21.9%	17.3%	12.7%	15.0%	15.6%

Figures on this table indicate the percentage of impact on the growth rate caused by each factor alone. In other words, if you total each column, they do not add up to 100%.

To calculate the relative importance of each factor, we prorate these figures so that each column adds up to 100%. Table 13 shows the relative contribution of each factor for different initial withdrawal rates. Figure 13 depicts the same in graphical format.

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We observe that, by far, the most important factor across all withdrawal rates is the sequence/volatility of returns. When withdrawal rates are below sustainable, then the next important factor is asset allocation. As withdrawal rates increase and portfolio life shortens, the importance of asset allocation diminishes and is replaced by the effect of inflation as the second most important factor.

At withdrawal rates larger than sustainable, the impact of the luck factor exceeds 50%. This leads us to conclude that:

1. if the sustenance of retirement income is your prime objective,
2. if withdrawal amounts are larger than sustainable[vii], and
3. if hoping for good luck is not your retirement strategy

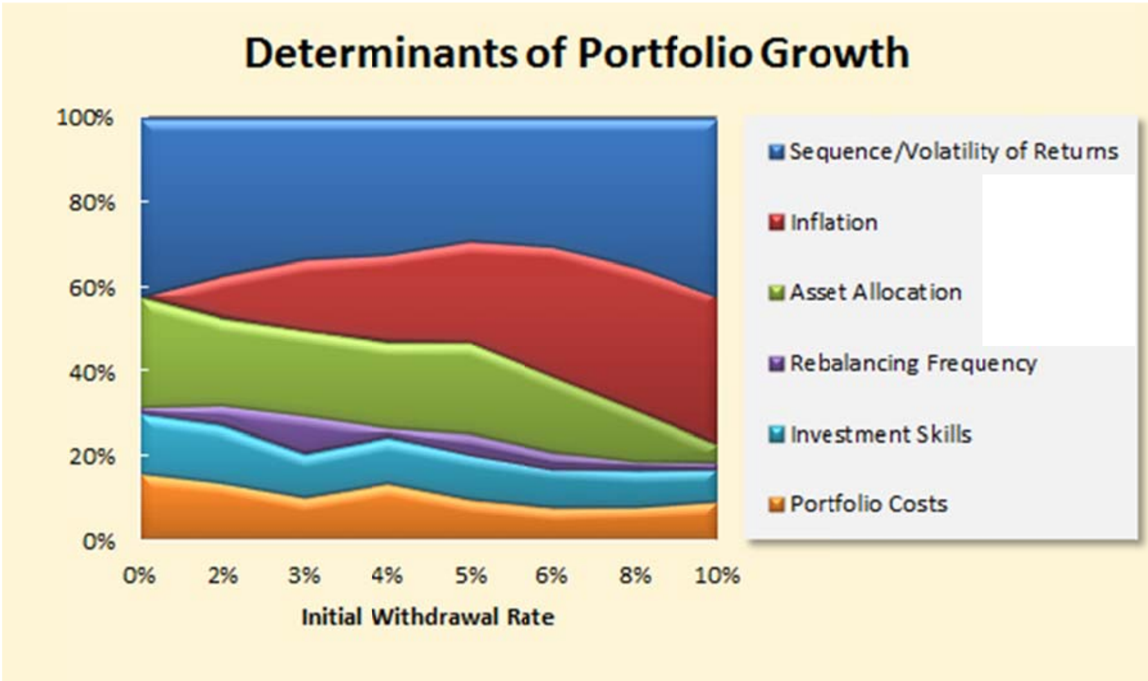
then don't expect that asset allocation to make much of a difference on the outcome. Our real problem is how to manage "luck" effectively.

Table 13: Determinants of a portfolio's growth for various withdrawal rates

	Initial Withdrawal Rate							
	0%	**2%**	**3%**	**4%**	**5%**	**6%**	**8%**	**10%**
	Determinants of Portfolio Growth							
Luck Factor:								
Sequence/Volatility of Returns	42%	37%	33%	32%	29%	30%	35%	41%
Inflation	0%	10%	17%	21%	24%	31%	34%	36%
Manageable Factors:								
Asset Allocation	26%	21%	20%	20%	21%	17%	11%	4%
Rebalancing	2%	5%	9%	2%	5%	4%	3%	1%
Fund Management Skills	14%	13%	11%	11%	11%	10%	8%	9%
Portfolio Costs	16%	14%	10%	14%	10%	8%	9%	9%

Note: All figures are rounded.

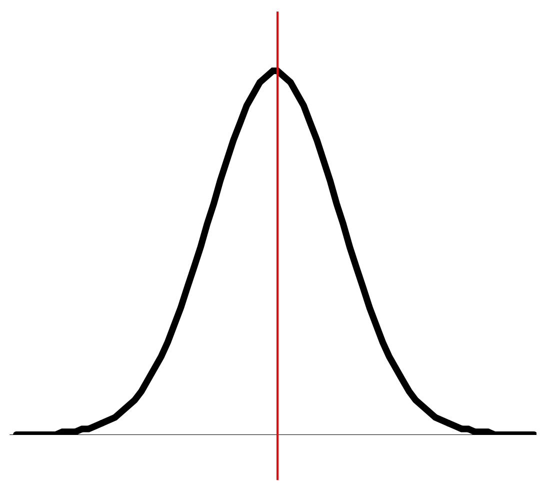
Figure 13: Determinants of a portfolio's growth for various withdrawal rates



Appendix A:
The Background of Gaussian Approach and its Shortfall

German mathematician and scientist Carl Friedrich Gauss published his monograph in 1809 that described several important statistical concepts, including what later evolved to "Gaussian distribution" or "Normal distribution" curve. It is also known as the "Bell Curve" because it resembles a church bell. The formula, or the chart resulting from this formula, describes the probability of occurrence (the vertical scale) of a random event or a random measurement. It indicates that most of the outcomes of a random event cluster around an "average". As you go further away from the average, then the chances of its occurrence drop exponentially.

Figure A1: The Gaussian distribution curve



The key word in the Gaussian universe is "random". When we study the monthly market behavior over the last century, we find that in about 94% of the time the equity index moves randomly, which we call "normal". Keep in mind; the word "normal" is used here in a mathematical sense. Events outside this "normal" are not "abnormal" or "deviant" or "new-normal"; they are just "extreme" in frequency and/or severity.

The other 6% of the time, the equity index does not fit into this definition of randomness: An upward trend begets a larger upward trend (more people start investing towards the end of a bull-market run) and a downward trend begets a larger downward trend (more people sell as the market drops precipitously).

So, in about 3% of the time, a portfolio loses significant proportion of its value. And in about 3% of the time it makes significant amount of money. These are the extreme events that happen much more frequently (and unexpectedly) than the Gaussian math suggests. The rest of the time, not much happens.

Figure A2: Depiction of Extremes and Normals

All Normal and Extreme Markets (DJIA – Index only):
"Buy-and-Hold": $1,000 invested in 1900 grows to about $158,000 at the end of 2009.

Extreme Losses (Left Fat Tail):
- Monthly losses larger than 10% occur in about 3% of the time
- It "feels like" an annualized loss of 86%
- If you miss extreme losses, $1,000 invested in 1900 grows to about $88 million at the end of 2009.

"Normal" Markets:
- Monthly fluctuations between +10% and -10% occur in about 94% of the time
- If you miss both extremes, $1,000 invested in 1900 grows to about $537,000 at the end of 2009.

Extreme Gains (Right Fat Tail):
- Monthly gains larger than 10% occur in about 3% of the time
- It "feels like" an annualized gain of 381%
- If you miss extreme gains, $1,000 invested in 1900 remains $1,000 at the end of 2009.

Since this type of market behavior does not match the Gaussian model, mathematicians in the financial industry thought that they can solve this problem by adding "fat-tails" to represent it better. Figure A3 depicts this.

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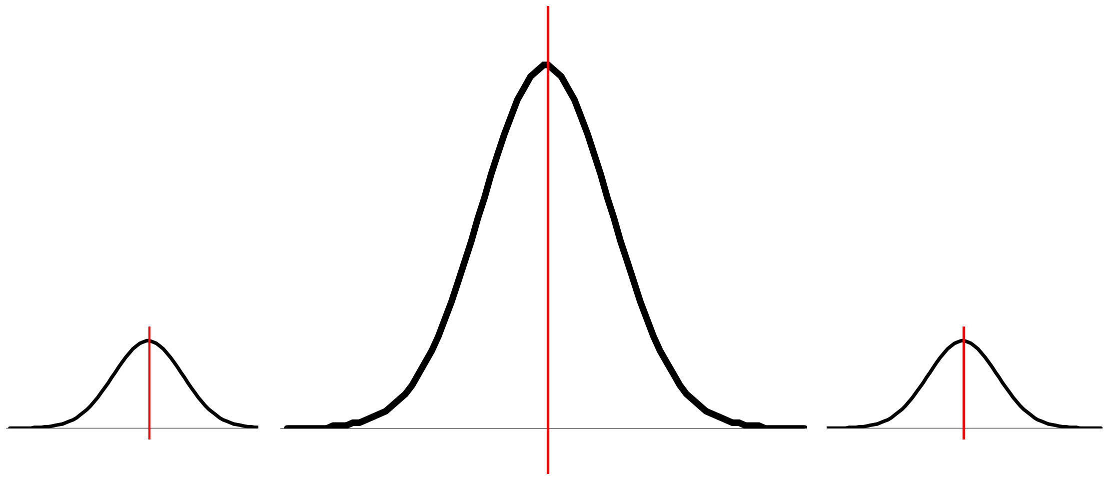
Figure A3: Depiction of Extremes and Normals in the Gaussian Math



By definition, the extreme events are rare and short-lived (3% of the time). Here is how the damage of sequence of returns happen: After a good bull-market run up in the right-fat tail region, the market index does not just slide back to the "normal" region, but it usually follows Newton's third law of physics: "To every action there is always an equal and opposite reaction". It moves all the way to the left fat-tail, causing great losses. It might bounce a few times back-and-forth between these two fat-tails like a table-tennis ball, creating great volatility in its path. In that, those who are last-in to the right tail are punished the most; they experience the largest losses. Eventually, the *volatility of returns* subsides and the market index moves to the "normal" region. In its wake, it often leaves an adverse *sequence of returns*.

History shows that given sufficient time, an accumulation portfolio eventually recovers from its losses. However, the definition of "sufficient time" covers a very wide spectrum depending on which regime this loss has occurred at the first place.

If the loss occurs in "normal" markets, then "sufficient time" for recovery can be from a couple of days to a couple of years. On the other hand, the recovery time increases exponentially if this loss occurs in extreme markets. Furthermore, the "math of loss" shows that it is significantly harder to recover in a distribution portfolio than in an accumulation portfolio[viii]. If the portfolio is already in distribution stage, or it is within 4 years (one market cycle) of the distribution stage, then a full recovery is unlikely to happen.

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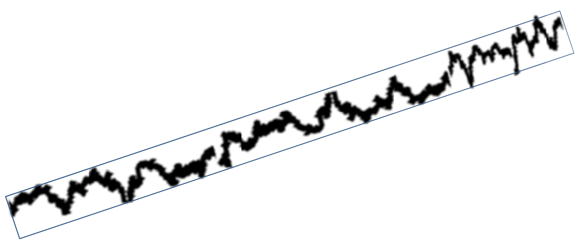
What is wrong with using fat-tails and the Gaussian math to simulate portfolios? You might be able to "curve-fit" the historical volatility of returns to some form of a distribution formula and we have no problem with that. However, you cannot model the patterns of sequence of returns (i.e. the severity and persistence of volatility of returns) with the Gaussian distribution formula. These patterns occur as a result of specific correlations between various economic factors such as equities, bond yields, interest rates and inflation in typical market cycles. Let's look at market trends that affect the sequence of returns:

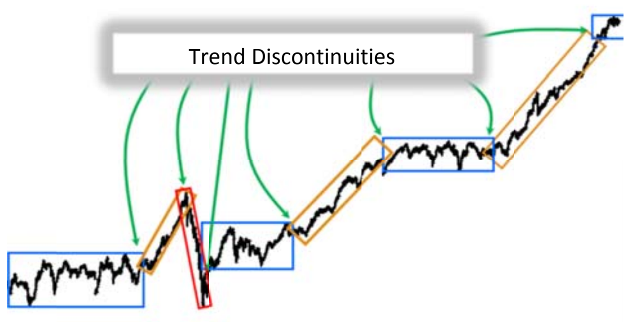
Secular Trends: Most Monte Carlo simulators use an average growth rate with some "standard deviation" attached to it, as depicted in Figure A4. In this case, the portfolio grows at about 8% per year, plus minus a random deviation from that average drawn on a semi-log scale.

Figure A4: Index value over time (on a semi-log scale) in a randomly generated simulation



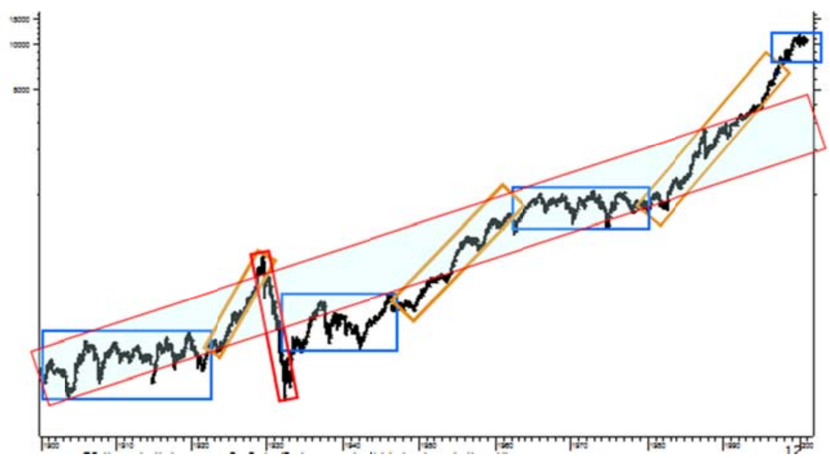
The reality is, in the long term, the equity market might be in a secular bullish trend lasting 10 to 20 years (1920-1929, 1949-1962, 1982-1999). Or, it might be in a secular sideways trend lasting as long as 20 years (1900-1920, 1938-1948, 1964-1982), as depicted in Figure A5. At each secular trend change, there is a "trend discontinuity".

Figure A5: Index value over time (on a semi-log scale) of actual market history starting in 1900



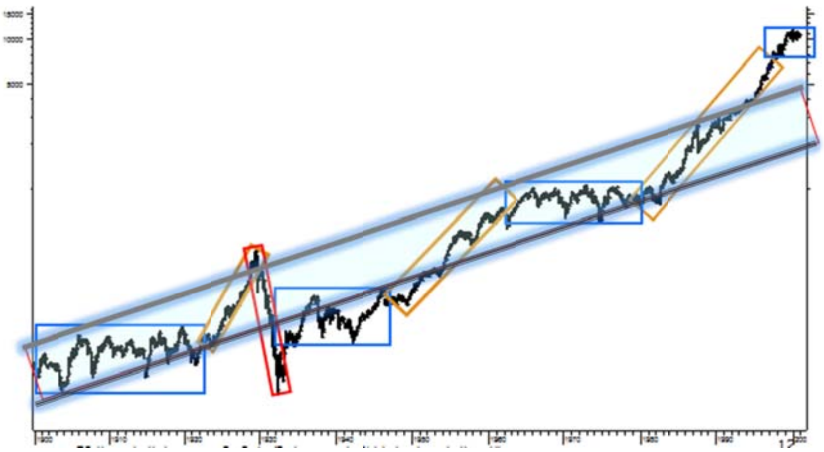
When you try to implement the Gaussian outcome to the market index then you would end up with something like in Figure A6. Here, we see clearly that the Gaussian model ignores all trend discontinuities and misses most of the market realities.

Figure A6: Overlapping Gaussian model and the market index (1900-2000)



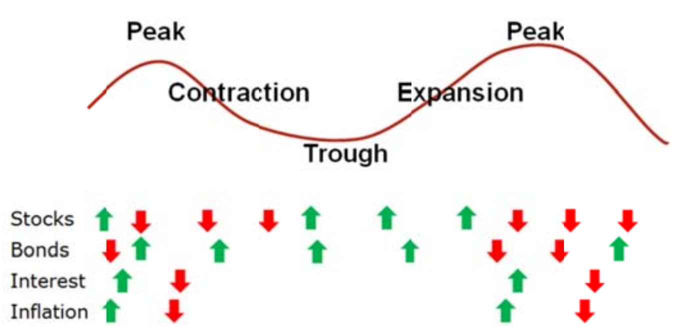
When you add fat-tails on the original distribution curve, then it merely increases the frequency of extremes, but it does not make the model any better. In Figure A7, the higher frequency of extremes with fat-tails is indicated by thicker top and bottom lines of the rectangle:

Figure A7: Overlapping Gaussian model (with fat-tails) and the market index (1900-2000)



Cyclical Trends: Each secular trend is made up of several cyclical trends which are generally a reflection of business cycles (see Figure A8). Within each cyclical trend, there is a specific **sequence of events**; which we define as "patterns of directions in inflation, interest rates, bond yields and equities with a specific array of inter-correlations".

Figure A8: Cyclical trends



There are four distinct phases and four distinct objects in this pattern. When randomized, there is a one in sixteen chance of modeling this particular sequence of events correctly. In other words, when you run sixteen thousand random simulations, only one thousand of these will likely have the correct pattern of up/down trends within a business cycle. The remaining fifteen thousand simulations will not only be in the wrong order but they will render the overall outcome useless.

Sometimes, when you run the Monte Carlo for accumulation portfolios, you might notice the unbounded, fanning-out effect of simulation lines. This definitely does not happen in real life. An extreme trend in one direction is eventually replaced with another extreme trend in the opposite direction until the excesses of both forces are expended. For these reasons, the effect of the sequence of returns is missed by man-made simulators that are based on Gaussian math.

In the early years of his research in the late 1990s, the author of this whitepaper developed a Monte Carlo simulator that included these three secular trends, each with its own time dimension. It was able to simulate these trend discontinuities reasonably well. However, it involved excessive data mining for each different market index for it to be reliable.

Philosophically, we rather observe and work with the raw data only. We do not want to create mathematical models with custom-made data mining and then try to convince ourselves and others that it “should” work. So, for this reason, we discarded[ix] it shortly after we created it in favor of aftcasting.

Aftcasting reflects the sequence of returns exactly as it happened in history. As opposed to forecasting, aftcasting is a method developed by the author for analyzing investment outcomes. It includes the actual historical equity performance, inflation rate and interest rate, as well as the actual historical sequencing of these data sets. There are no mathematical formulas, complicated procedures, assumptions or limitations. It merely displays what would have happened in history with no room for contaminated data or misguided interpretations of outcome.

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About Aftcast.com

Aftcast.com provides research to its clients in the area of distributions. The research is based on non-Gaussian philosophy using actual market history. It helps its clients to better understand the behavior and impact of various distribution strategies.

This report was researched and authored by Jim Otar, CFP, CMT, BASc, MEng, who is the founder of aftcast.com.

For your comments and feedback, or to learn more about aftcasting, please visit www.aftcast.com or send an email to jim@retirementoptimizer.com

[i] Otar, Jim, "Unveiling the Retirement Myth", 2009

[ii] Otar, Jim, "High Expectations and False Dreams", 2001

[iii] The annual data of the Standard and Poor Composite Stock Price Index started in 1871. S&P index was established in 1926, including 90 large cap stocks and later in 1957 it was changed to hold 500 stocks. In our calculations, we used the annual index data from the tables in Chapter 26 of Robert J. Shiller's book "Market Volatility", the MIT Press, ISBN 978 0262 691512" and from the Standards and Poor's Public website, https://www.sp-indexdata.com

[iv] Annual inflation: U.S. Bureau of Labor Statistics, wholesale price index for the years between 1900–1913, the consumer price index after 1913

[v] Interest Rate: 1900–1987, courtesy of "Market Volatility", by Robert J. Schiller, MIT Press, [1997], page 440–441, data series 4, recent history from mortgage-x, mortgage information service, http://mortgage-x.com

The fixed income rate used in this analysis is the historical 6-month CD rate plus 0.5% yield premium, net after portfolio expenses. This represents approximately a bond portfolio with about five to six-year average maturity, held until maturity (no capital gains/losses)

[vi] Disclosure: The author of this paper spent the first twenty years of his life where inflation varied between 8% and 25% per year.

[vii] We currently calculate the sustainable withdrawal rate to be about 3.3% at age 60, 3.6% at age 65, 4% at age 70, 4.5% at age 75 if the plan is designed to provide income until the age at which the probability of survival is 10% or less.

[viii] Otar, Jim C., 2009, "Unveiling the Retirement Myth", pages 90 -105, Chapter 8, "Mathematics of Loss"

[ix] for the curious, it is still available for downloading at www.retirementoptimizer.com